MA4553: A MATHEMATICAL INTRODUCTION TO IMAGE PROCESSING AND ANALYSIS, WITH SOME SURPRISING APPLICATIONS

Effective Term
Semester A 2024/25

Part I Course Overview

Course Title
A Mathematical Introduction to Image Processing and Analysis, with Some Surprising Applications

Subject Code
MA - Mathematics
Course Number
4553

Academic Unit
Mathematics (MA)

College/School
College of Science (SI)

Course Duration
One Semester

Credit Units
3

Level
B1, B2, B3, B4 - Bachelor's Degree

Medium of Instruction
English

Medium of Assessment
English

Prerequisites
Nil

Precursors
Nil

Equivalent Courses
Nil

Exclusive Courses
Nil

Part II Course Details

Abstract

The course introduces to digital images, their mathematical structure, and to the mathematical theories that explain how images are perceived and how they can be automatically analysed and modified. The course will use and present in self-contained way mathematical tools such as basic Fourier analysis, a few classic linear partial differential equations, and discrete probability. Their presentation, being specific of images, will be self-contained. Each lecture of the course ends up in a description of a practical powerful algorithm to process images.

We start with the question of linking by Fourier analysis the discrete object (the digital image) to its continuous representation, that enables the use of mathematical operators.

We continue with the theory and algorithms for image resampling, color and contrast manipulation, image retouching. Then the theory for invariant image representation, shape recognition, and automatic image comparison. The students will be invited to practice these algorithms by themselves on their own images.

Course Intended Learning Outcomes (CILOs)

	CILOs	Weighting (if app.)	DEC-A1	DEC-A2	DEC-A3
1	Identify the mathematical structure of images and how it relates to our perception of them.	20	x	x	
2	Develop a full understanding of fundamental image processing algorithms and acquire the ability to interpret visually their effects and defaults.	20	x	x	x
3	Become able to develop an image processing algorithm with some specification.	20	x	x	x
4	Acquire basics of discrete Fourier analysis and its use to solve PDEs and perform geometric transforms on images. (There is no prerequisite, the presentation will be self-contained)	20	x	x	
5	Learn to resolve and use classic partial differential equations such as the heat and the Poisson equation on rectangular domains, applications to image processing. (There is no prerequisite, the presentation will be self-contained).	20	x	x	x

A1: Attitude
Develop an attitude of discovery/innovation/creativity, as demonstrated by students possessing a strong sense of curiosity, asking questions actively, challenging assumptions or engaging in inquiry together with teachers.

A2: Ability
Develop the ability/skill needed to discover/innovate/create, as demonstrated by students possessing critical thinking skills to assess ideas, acquiring research skills, synthesizing knowledge across disciplines or applying academic knowledge to real-life problems.

A3: Accomplishments
Demonstrate accomplishment of discovery/innovation/creativity through producing /constructing creative works/new artefacts, effective solutions to real-life problems or new processes.

Learning and Teaching Activities (LTAs)

	LTAs	Brief Description	CILO No.	Hours/week (if applicable)
1	Lecture	Learning through teaching is primarily based on lectures.	1, 2, 3, 4, 5	39 hours in total
2	Take-home assignments	Students will be recommended to solve simple maths exercises after each course, to understand in depth the mathematical techniques involved in the course.	1, 2, 3, 4, 5	after-class
3	Online applications	Students will be required to perform online tests of the main algorithms being taught, on images of their own.	1, 2, 3, 4, 5	after-class
4	Math Help Centre	Learning activities in Math Help Centre provides students extra help.	1, 2, 3, 4, 5	after-class

Assessment Tasks / Activities (ATs)

	ATs	CILO No.	Weighting (%)	Remarks (e.g. Parameter for GenAI use)
1	Quizzes/Test/Midterm	1, 2, 3, 4, 5	15	Questions are designed for the first part of the course to see how well the students have learned the basic algorithms, their theory, and link them to applications.
2	Experimental reports on online experiments	1, 2, 3, 4, 5	10	Delivery of an experimental report on the main algorithms presented in the course. Experiments will be performed online at www.ipol.im or based on simple codes written by the student.
3	Formative take-home maths exercises	1, 2, 3, 4, 5	15	The goal is to write down quick solutions of exercises given in the lecture notes.

Continuous Assessment (%)

40

Examination (%)

60

Examination Duration (Hours)

2

Additional Information for ATs

For a student to pass the course, at least 30% of the maximum mark for the examination must be obtained.

Assessment Rubrics (AR)

Assessment Task

1. Quizzes/Test/Midterm

Criterion

1.1 Understanding of the mathematical proofs
1.2 Ability to describe an image processing algorithm
1.3 Ability to comment the visual effects of an image processing algorithm

Excellent (A+, A, A-)

ability to solve quickly and perfectly simple illustrating the main mathematical arguments of the course.

Good (B+, B, B-)

Ability to solve simple exercises illustrating the main mathematical arguments of the course.

Fair (C+, C, C-)

Moderate ability to solve simple exercises illustrating the main mathematical arguments of the course.

Marginal (D)

Basic to insufficient ability to solve some simple exercises illustrating the main mathematical arguments of the course.

Failure (F)

Unable to solve simple exercises illustrating the main mathematical arguments of the course.

Assessment Task

2. Experimental reports on online experiments

Criterion

2.1 Ability to choose adequate images for experiments
2.2 Ability to explore quickly a new algorithm by adequate experiments
2.3 Ability to detect and comment on visual defaults caused by algorithms

Excellent (A+, A, A-)

Deep understanding of algorithms and of their effect on well chosen images

Good (B+, B, B-)

Good understanding of algorithms and of their effect on well chosen images

Fair (C+, C, C-)

Moderate understanding of algorithms and of their effect on images

Marginal (D)

Little understanding of algorithms and of their effect on images

Failure (F)

No understanding of algorithms and of their effect on images

Assessment Task

3. Examination

Criterion

3.1 Ability to make a variant of mathematical arguments seen in the course processing effect
3.2 Ability to conceive and describe precisely an algorithm with a prescribed image processing effect
3.3 Ability to analyse the visual content of an image and to link it to mathematical operators

Excellent (A+, A, A-)

Quick and well redacted solution of exercises, precise insightful description of algorithms and of their effects on images

Good (B+, B, B-)

Good solution of many exercises, precise description of algorithms and of their effects on images

Fair (C+, C, C-)

Acceptable solution of sufficiently many exercises, acceptable description of algorithms and of their effects on images

Marginal (D)

Insufficient solution of exercises, partial description of algorithms and of their effects on images

Failure (F)

Few or failed solution of exercises, insufficient description of algorithms and of their effects on images

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Assessment Task

4. Formative take-home maths exercises

Criterion

4.1 Ability to describe the solution of an exercise

Excellent (A+, A, A-)

All exercises are well solved and redacted with no time delay

Good (B+, B, B-)

Most exercises are well solved, redaction variable

Fair (C+, C, C-)

A majority of exercises are solved, redaction acceptable

Marginal (D)

Some exercises are solved, redaction sometimes wrong

Failure (F)

Few exercises solved, redaction insufficient or wrong

Part III Other Information

Keyword Syllabus

Images from discrete to continuous and vice versa: image representation using the Fourier-Shannon method, applications to geometric image transformations and their fast algorithms

Color and contrast: image dynamics, histogram equalization and its applications

Retinex theories of contrast perception and their applications

Solving linear partial differential equations in images and applications:

- Poisson editing: how to cut and paste in images, and many application
- Exact implementation of the heat equation: the scale space and its discretization

Pattern recognition invariance and the SIFT method, comparing automatically any two images and finding their common shapes

Geometric invariance in pattern recognition: affine geometry and the ASIFT method

This list is not exhaustive and additional topics may include the Wiener theory of image restoration, denoising and deblurring algorithms, etc.

Reading List

Compulsory Readings

	Title
1	Complete lecture notes provided by the lecturer (can be updated during the course)
2	Complete slides provided by the lecturer (can be updated during the course)
3	The IPOL online executable papers (www.ipol.im) directly linked to one chapter of the course

Additional Readings

	Title
1	J.M. Morel and G. Yu: is SIFT scale invariant? IPI 2011
2	R. C. Gonzalez, R.E. Woods, Digital Image Processing, Prentice Hall (3rd edition, 2008)
3	D. Lowe Object recognition from local scale-invariant features IJCV 2004
4	P. Pérez, M. Gangnet, A. Blake, Poisson image editing ACM TOG 2003