MA4551: INTRODUCTION TO FUNCTIONAL ANALYSIS

Effective Term
Semester A 2022/23

Part I Course Overview

Course Title
Introduction to Functional Analysis

Subject Code
MA - Mathematics
Course Number
4551

Academic Unit
Mathematics (MA)

College/School
College of Science (SI)

Course Duration
One Semester

Credit Units
3

Level
B1, B2, B3, B4 - Bachelor's Degree

Medium of Instruction
English

Medium of Assessment
English

Prerequisites
MA2503 Linear Algebra / MA1503 Linear Algebra with Applications, and
MA3524 Analysis / MA3526 Analysis

Precursors
Nil

Equivalent Courses
Nil

Exclusive Courses
Nil

Part II Course Details

Abstract

As a branch of mathematical analysis, functional analysis brought with it a change of focus from the study of functions on the Euclid space to the analysis of abstract infinite-dimensional spaces, for example, Banach spaces and Hilbert spaces. As such it established a key framework for the development of modern analysis, and is particularly useful for the study of differential and integral equations.

This elective course will introduce the basic concepts of the functional analysis, such as the normed linear space, duality, weak convergence, and the Hilbert space. The basic knowledge of linear algebra and elementary analysis is required. The course will also discuss several fundamental and important theorems, for example, the Hahn-Banach theorem, the Riesz-Frechet representation theorem, and the fixed point theorems. These topics present the basic structure of this subject, and loom large in the body of mathematics. Finally, there will be a careful balance between the theories and the applications. The students, who complete this course, are expected to be prepared for the advanced Math courses at the graduate level and for the modern development of the research in the analysis.

Course Intended Learning Outcomes (CILOs)

	CILOs	Weighting (if app.)	DEC-A1	DEC-A2	DEC-A3
1	Introduce the normed linear space and the Hahn-Banach theorem.	10		x	
2	Introduce the bounded linear functionals, the applications of duality, and the weak convergence	30		x	
3	Introduce the Hilbert space, the Riesz-Frechet representation theorem, the Lax-Milgram lemma, and applications	30	x	x	
4	Fixed point theorems, and solve the problems arising from the analysis	30	x	x	x

A1: Attitude
Develop an attitude of discovery/innovation/creativity, as demonstrated by students possessing a strong sense of curiosity, asking questions actively, challenging assumptions or engaging in inquiry together with teachers.

A2: Ability
Develop the ability/skill needed to discover/innovate/create, as demonstrated by students possessing critical thinking skills to assess ideas, acquiring research skills, synthesizing knowledge across disciplines or applying academic knowledge to real-life problems.

A3: Accomplishments
Demonstrate accomplishment of discovery/innovation/creativity through producing /constructing creative works/new artefacts, effective solutions to real-life problems or new processes.

Teaching and Learning Activities (TLAs)

	TLAs	Brief Description	CILO No.	Hours/week (if applicable)
1	Lectures	Learning through teaching is primarily based on lectures	1, 2, 3, 4	39 hours in total

2	Assignment	Learning through assignments helps students understand basic concepts and several fundamental and important theorems in the functional analysis.	1, 2, 3, 4	After-class

Assessment Tasks / Activities (ATs)

	ATs	CILO No.	Weighting (%)	Remarks (e.g. Parameter for GenAI use)
1	Test	1, 2, 3	20	
2	Hand-in assignments (3 or above)	1, 2, 3, 4	20	

Continuous Assessment (%)

40

Examination (%)

60

Examination Duration (Hours)

2

Additional Information for ATs

40% Coursework
60% Examination (Duration: 2 hours, at the end of the semester)
For a student to pass the course, at least 30% of the maximum mark for the examination must be obtained.

Assessment Rubrics (AR)

Assessment Task

1. Test

Criterion

Ability in problem solving

Excellent (A+, A, A-)

High

Good (B+, B, B-)

Significant

Fair (C+, C, C-)

Moderate

Marginal (D)

Basic

Failure (F)

Not even reaching marginal levels

Assessment Task
2. Assignments

Criterion
Understanding of concepts and applications

Excellent (A+, A, A-)
High

Good (B+, B, B-)
Significant

Fair (C+, C, C-)
Moderate

Marginal (D)
Basic

Failure (F)
Not even reaching marginal levels

Assessment Task
3. Examination

Criterion
Comprehensive ability in independent problem solving

Excellent (A+, A, A-)
High

Good (B+, B, B-)
Significant

Fair (C+, C, C-)
Moderate

Marginal (D)
Basic

Failure (F)
Not even reaching marginal levels

Part III Other Information

Keyword Syllabus
Normed linear space, duality, weak convergence, Hilbert space, the Hahn-Banach theorem, the Riesz-Frechet representation theorem, and the fixed point theorem

Reading List

Compulsory Readings

	Title
1	P. Lax, 2002, Functional Analysis. Wiley-Interscience.
2	Y. Eidelman, V. Milman & A. Tsolomitis, 2004, Functional Analysis: An Introduction (Graduate Studies in Mathematics) . American Mathematical Society.
3	D. Xia, Z. Wu, S. Yan & W. Shu, 2010, Real Analysis and Functional Analysis (2d edition, in Chinese). Higher Education Press.
4	Lecture notes distributed in class

Additional Readings

	Title
1	Yosida, K., Functional Analysis, Springer-Verlag, 6th edition, 1980
2	Conway, J. B.: A Course in Functional Analysis, 2nd edition, Springer-Verlag, 1994