CESifo WORKING PAPERS

8745
2020

December 2020

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CESifo

Impressum:

CESifo Working Papers
ISSN 2364-1428 (electronic version)
Publisher and distributor: Munich Society for the Promotion of Economic Research - CESifo GmbH
The international platform of Ludwigs-Maximilians University's Center for Economic Studies and the ifo Institute
Poschingerstr. 5, 81679 Munich, Germany
Telephone +49 (0)89 2180-2740, Telefax +49 (0)89 2180-17845, email office@cesifo.de
Editor: Clemens Fuest
https://www.cesifo.org/en/wp
An electronic version of the paper may be downloaded
· from the SSRN website: www.SSRN.com
· from the RePEc website: www.RePEc.org
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CESifo *Working Paper No. 8745*

Do We Still Need Carbon-Intensive Capital When Transitioning to a Green Economy?

Abstract

This paper presents a two-sector green endogenous growth model to explore a mechanism that explains why carbon-intensive capital is not necessarily shut down during transition to a green economy. Without accumulating clean capital to offset carbon emissions, a tightening of climate regulation leads to the running down of carbon-intensive capital. However, if climate regulations induce stepping-up of carbon-free capital to offset warming damages, the economic value of carbon-intensive capital can be protected and the running down of carbon-intensive assets can be mitigated. The use of carbon-intensive capital gives the economic means to enhance clean capital accumulation and sustain endogenous growth. Both carbon-intensive and carbon-free capital may thus be needed for an efficient transition to green growth.

JEL-Codes: Q540, Q430, Q320, O130, O440, C610.

Keywords: endogenous growth, green growth, two-sector growth model, climate policy.

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November 2020
We thank Francesca Diluiso, Christos Karydas, Jianlei Mo, Steven Poelhekke, Chiara Colesanti Senni, Shiba Suzuki, Frank Venmans, Sophie Zhou and participants at the EAERE 2020 Annual Conference, the SURED 2020 conference and various seminars for their helpful comments and suggestions.

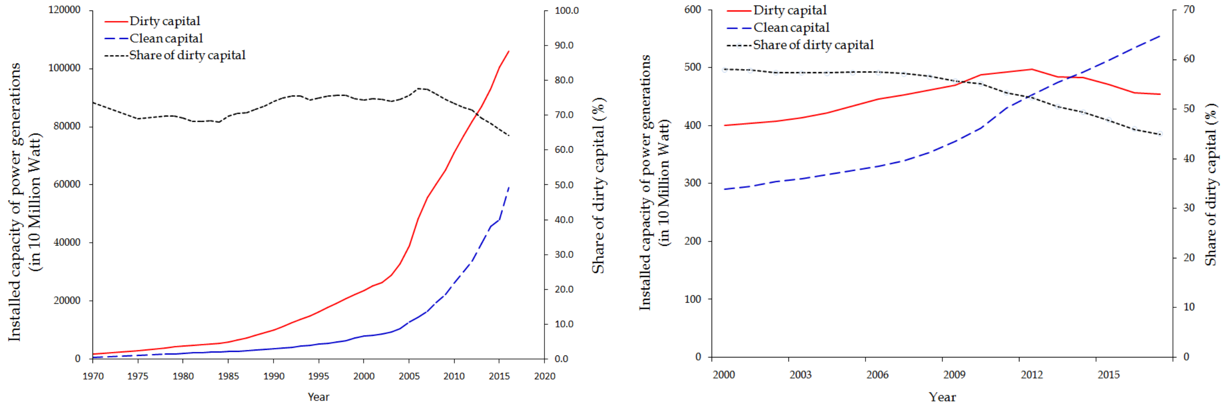
1 Introduction

Our conjecture is that production with carbon-intensive dirty capital may be needed for quite some time along an endogenous growth path even if climate policies are implemented to regulate carbon pollution. The reason is that this type of production provides the economic means to build up more rapidly clean carbon-free capital for offsetting warming damages and accelerating the transition to green growth.[1] It is this mechanism that can postpone or avoid the running down of carbon-intensive dirty capital in the transition to green growth. Factors such as technology and policy shocks can induce demand shifts towards carbon-free renewables and replace carbon-intensive fossil fuels. Carbon-intensive dirty resource and capital would thus be at risk of becoming stranded assets and suffer from premature write-downs and devaluations (McGlade and Ekins, 2015; Baldwin et al., 2020; van der Ploeg and Rezai, 2020a,b). In this context, exploring a mechanism that might explain why carbon-intensive capital is not run down during a green transition is particularly important, because addressing this issue gives important insights into protecting revenues, wealth, and employment in fossil fuel-rich countries and their transition to a green economy.

The existing studies of stranded assets focus on identifying the determinants of assets stranding such as tightening of climate policies (Rozenberg et al., 2018; van der Ploeg and Rezai, 2020a), the probability of a breakthrough and consequent drastic reduction in the costs of renewable technology (Jaakkola and van der Ploeg, 2019), investment irreversibility and adjustment costs (Baldwin et al., 2020), and lobbying and political economy (Kalkuhl et al., 2019). Our work differs from the existing literature by exploring a potential mechanism for why carbon-intensive capital assets might not need to be run down. We show that if investments in clean capital can be stepped up sufficiently to offset emissions and warming damages caused by carbon-intensive capital, the economic values of dirty capital can be protected and carbon-intensive assets can still play a role in the green transition. Climate regulation might induce stepping-up of investment in carbon-free clean capital to offset warming damages and can thus create an endogenous growth outcome. Along the endogenous growth path, both carbon-intensive and clean capital are thus accumulated simultaneously.

Our conjecture is supported by the stylized facts in Figure 1. When China's investments in carbon-free clean capital, as measured by installed capacities of power generation using low-carbon energy such as solar and wind, kick off over 1970-2016, the stock of carbon-intensive capital, as measured by power generation using fossil energy such as coal and oil, grows at the same pace and does not fall precipitously. Even when stringent regulations are put in place to curb pollution around 2005, the stock of carbon-intensive capital is still expanding although its share in total capital shows signs of decline. A similar

[1] This paper conceptualizes carbon-free clean capital as a broad measure of capital such as the equipment based on renewable energy, clean production facilities, high-efficiency, low-emission plants, climate geoengineering, carbon capture and storage(CCS), and negative emission technology.



(a) China **(b)** European Union

Figure 1: The Historical Trend of Carbon-intensive and Clean Capital. Panel (a) plots capital accumulation in China over 1970-2016. Carbon-intensive capital is augmented at the same pace as clean capital. Panel (b) plots capital accumulation in the EU over 2000-2017: carbon-intensive capital accumulation slows down but the stock keeps up without sudden drop in its use. Carbon-intensive and clean capital are measured by the installed capacity of power generation using fossil energy (coal, oil, and natural gas) and low-carbon energy (solar, wind, hydropower, and nuclear), respectively. The data is collected from *Statistics of China Electric Power Industry 2017* (China Electric Power Press, 2017b) and *Eurostat regional yearbook, 2019* (European Commission, 2019).

trend is seen in the EU market, where the growth of carbon-intensive capital slows down but the installed capacity keeps up without a sudden sharp drop of assets of companies using such carbon-intensive capital.

Accordingly, the transition to a green economy might accommodate a potential mix of carbon-intensive and clean capital without curbing carbon-intensive capital as in van der Ploeg (2016) or in Hambel et al. (2020) using diversification arguments. If ambitious climate policies are enacted, deployment of clean capital could be induced to offset the warming damages caused by carbon-intensive capital. In this case, both carbon-intensive (e.g., fossil energy) and carbon-free clean capital (e.g., renewable energy, high-efficiency and low-emission production, and carbon control technology) can coexist, and the clean capital acts as a potential enabler of utilizing otherwise stranded carbon-intensive capital. In other words, carbon-free capital, when widely deployed, acts to decarbonize carbon-intensive capital and break the link between carbon-intensive capital and global warming damages.[2]

We thus present a two-sector green endogenous growth model which describes the interplay between carbon-intensive capital, carbon-free capital, carbon emission, temperature, and warming damages. The model is used to explore a potential mechanism that might avoid the running down of carbon-intensive cap-

[2]Climate geoengineering and CCS could be attractive and inexpensive relative to mitigation options if stringent carbon pricing for the 2°C target is put in place. It will also help drive CCS deployment to rescue stranded assets if the value losses of stranded carbon assets outweigh the added costs of CCS (e.g., Anderson and Newell, 2004; Clark and Herzog, 2014; IEA, 2017; Heutel et al., 2016, 2018).

ital. We show that a tightening of climate regulation to correct for warming damages leads to the running down of carbon-intensive dirty capital as in McGlade and Ekins (2015), Baldwin et al. (2020) and van der Ploeg and Rezai (2020a,b). However, this running down would becomes less severe if the accumulation of carbon-free capital can be stepped up to offset the effects of carbon-intensive capital on warming damages. More specifically, carbon-free capital offsets global warming damages caused by carbon-intensive capital, protects its economic values, and avoids the need to run down carbon-intensive capital. In the meantime, carrying on with the use of carbon-intensive capital gives the economic means to enhance clean capital accumulation via investment and intersectoral capital reallocation. This effect thus provides a mechanism through which both carbon-intensive and carbon-free capital is needed for a transition to green growth. Tightening of climate regulation, by inducing carbon-free capital to offset carbon emissions and warning damages, does not necessarily give rise to the need to run down the stock of carbon-intensive dirty capital.

A crucial feature of our argument is that the above-mentioned interplay between carbon-intensive and carbon-free capital leads to an endogenously growing economy, where both carbon-intensive and carbon-free capital, consumption, and investment can sustain growth over time. The augmented stock of carbon-free capital, by offsetting emissions caused by carbon-intensive capital, curbs the growth in carbon emissions and facilitates the move towards a green growth outcome. Furthermore, the mechanism of avoiding the running down of carbon-intensive capital through clean capital works for endogenous growth under both stock-related climate damages (i.e., warming damages are attributable to the stock of cumulative carbon emissions) and flow or rate-related climate damages (i.e., warming damages are attributable to the flow of carbon emissions). The primary difference between these two types of damages is that the economy subject to stock pollution damages needs to accumulate more clean capital and less carbon-intensive ones for generating green endogenous growth. Also, tightening regulations to correct for stock rather than flow climate damages reduces the equilibrium amount of consumption, investment, and outputs.

To verify our conjecture, we construct a unique dataset for 27 countries combining OECD and World Bank data, and employ a panel vector autoregression (PVAR) model to empirically examine the potential movements of carbon-free and carbon-intensive capital over time. Our empirical results show that an increase in carbon-free capital is associated with an increase in carbon-intensive dirty capital. This positive association is reinforced by stringent environmental regulation as this encourages the development and deployment of carbon-free clean capital.

This paper is related to the literature on stranded assets and climate policy. Bretschger and Soretz (2018) examine how stochastic policy shocks could affect investors' decisions, capital valuation, and stranded assets. Baldwin et al. (2020) show that irreversibility in dirty capital implies an earlier shift to investment in the clean sector, which avoids a future stranding of assets in the fossil-fuel-based sector. Sen and von Schickfus (2020) use a careful event-study analysis to empirically examine whether the market valuation of companies owning fossil fuel assets has priced in the risk of stranding assets due to unanticipated climate regulation. Rozenberg et al. (2018) analyze how using alternative policy instruments (e.g., first-best carbon prices, and second-best feebates or standards) leads to different dynamic transitions to clean capital and premature retirement of existing carbon-intensive capital. Kalkuhl et al. (2019) show that stranded assets occur due to unanticipated time-inconsistent climate policy. Time-inconsistency, in turn, results from the government taking political economy aspects into account. van der Ploeg and Rezai (2020a) show that with a risk of policy tipping or an unanticipated change in future climate policy, past investments in exploration capital become stranded and the share prices of oil and gas companies will drop. Barnett (2019) shows that the risk of future climate policy actions accelerates extraction, strands oil reserves, and generates a run on oil. Hambel et al. (2020) establish that the process of transition towards a zero-emission economy should be stopped if there is a balance between green and dirty capital, and show that diversification considerations prevent the agent from driving the carbon-intensive dirty capital stock to zero.

Our work is also related to the green growth literature and the Environmental Kuznets Curve literature (e.g., Grossman and Krueger, 1995; Stern and Common, 2001). Such studies explore the mechanisms for green growth through channels such as pollution abatements (e.g., Hartman and Kwon, 2005; Brock and Taylor, 2010), pollution-augmenting endogenous technical change (e.g., Peretto, 2009; Acemoglu et al., 2012; Bretschger and Smulders, 2012; Bretschger et al., 2017), and substitution between carbon-intensive fossil energy to low-carbon renewable backstops (e.g., Tahvonen and Salo, 2001; Smulders et al., 2012; van der Ploeg and Withagen, 2012; van der Ploeg and Withagen, 2014). Our focus, in contrast, is on the mechanism for green endogenous growth through the interplay between capital accumulation and climate regulations.

Section 2 presents our green growth model. Section 3 investigates the effect of climate policy on the time paths of carbon-free and carbon-intensive capital. Section 4 explores the mechanism of green endogenous growth with simultaneous accumulation of both carbon-intensive and clean capital. Section 5 gives some empirical support for this mechanism. Section 6 concludes.

2 A Green Uzawa-Lucas Growth Model

2.1 The Setup

We build on and extend the Uzawa-Lucas endogenous growth model (e.g., Uzawa, 1965; Lucas, 1988; Mulligan and Sala-i-Martin, 1992; Caballe and Santos, 1993) to allow for carbon-intensive capital, carbon-free capital, carbon emissions and global warming. The economy has a representative firm using carbon-intensive and clean capital to produce final goods according to the production technology

$$Y(t)=E(t)^{-\gamma}K_D(t)^{\theta}K_C(t)^{1-\theta}, \tag{1}$$

where $Y(t)$, $K_D(t)$, $K_C(t)$ and $E(t)$ are aggregate production, the stock of carbon-intensive capital, the stock of clean capital, and the flow of pollution emissions at time t, respectively.[3] Here θ is the Cobb-Douglas weight of dirty capital in value added. The term $E^{-\gamma}$ captures how much total factor productivity is curbed in response to pollution emissions, and the effect of pollution on productivity is governed by the elasticity γ as in Graff Zivin and Neidell (2012), Chang et al. (2016) and Chang et al. (2019).

A growing body of climate change literature such as Mendelsohn (2000), Patt et al. (2010), Shayegh et al. (2016) and Michaelis and Wirths (2020) indicates the importance of flow- or rate-related climate damages, i.e., the rate of temperature increase (the speed of warming) or equivalently the flow of carbon emissions is a key driver of climate damages. This is motivated by empirical evidence that the rate of temperature increase has impacts on biodiversity and ecosystems (e.g., Leemans and Eickhout, 2004), agriculture (e.g., Lobell et al., 2008; Panda, 2018), forestry (e.g., Zhu et al., 2012; Pedlar et al., 2012), and long-lived capital and infrastructure (e.g., Mendelsohn, 2000; Patt et al., 2010). Below we provide the benchmark model where warming damages depend on temperature or equivalently cumulative emissions of carbon. We also analyze a version of our model where warming damages depend on the rate of temperature increase or equivalently the flow of carbon emissions (see Appendix A.6 for details).

The flow emissions of carbon pollutants are proportional to the stock of carbon-intensive dirty capital, i.e.,

$$E(t)=m(t)K_D(t), \tag{2}$$

where the intensity of carbon emissions is given by

$$m(t)=\psi Y(t)^{-1}, \tag{3}$$

and ψ is an emission intensity parameter. The intensity of carbon emission m is inversely related to the output Y in line with empirical evidence that supports a declining emission intensity.[4] Higher production is more likely to generate the spillover effect and thus lead to the decline of emission intensity as in Romer (1986).

We proceed by substituting (1) and (3) into (2), and derive the level of emissions as: $E(t)=\psi K_D(t)Y(t)^{-1}=\psi E(t)^{\gamma}K_D(t)^{1-\theta}K_C(t)^{\theta-1}$. The emission function can thus be simplified as

$$E(t)=P(K_D(t),K_C(t))=\xi K_D(t)^{\beta}K_C(t)^{-\beta}, \tag{4}$$

[3] K can be thought of as a broad measure of capital including physical capital and knowledge capital. So, K_D denotes the capital deployed in the carbon-intensive sector (e.g., oil and petroleum industries, steel, or cement) and K_C is the capital in the clean sector (e.g., carbon-free service sectors). In this case, stranding of dirty capital means that both tangible physical capital and intangible knowledge embodied in tangible physical capital will suffer from premature write-downs and devaluations.

[4] Specifically, we plot the data of China's carbon emission intensity versus GDP. The curve fitting can be produced using the inverse proportional function (3), $m(t)=\psi Y(t)^{-1}$, where $m(t)$ and $Y(t)$ are carbon emission intensity and GDP at time t, respectively. See Section 4 for the details.

where $\xi := \psi^{\frac{1}{1-\gamma}}$ and $\beta := \frac{1-\theta}{1-\gamma}$. Emissions are homogenous of degree zero with respect to K_C and K_D, meaning that there will be no growth in emissions when both carbon-intensive and clean capital are accumulated at the same rate. This feature is crucial for generating green endogenous growth and avoiding the running down of carbon-intensive capital. Furthermore, substituting (4) into (1) to replace $E^{-\gamma}$, the production function becomes

$$Y(t) = F(K_D(t), K_C(t)) = \eta K_D(t)^{\alpha} K_C(t)^{1-\alpha}, \tag{5}$$

where $\eta := \psi^{\frac{-\gamma}{1-\gamma}}$ and $\alpha := \theta - \frac{\gamma(1-\theta)}{1-\gamma}$. The marginal products of capital are positive and diminishing. Equation (5) implies cooperant production factors, i.e., $F_{K_D K_C} > 0$, so that clean capital increases the marginal product of dirty capital (e.g., by cleaning hazardous wastes to avoid erosion and deterioration). We assume imperfect substitution between carbon-intensive and clean capital as in the literature of environmental macroeconomics and green growth (e.g., Tahvonen and Salo, 2001; Tsur and Zemel, 2005; Acemoglu et al., 2012; Long, 2014; van der Meijden, 2014).

Following Dietz and Venmans (2019), our model keeps track of temperature change which evolves according to

$$\dot{T}(t) = \epsilon(\zeta S(t) - T(t)), \tag{6}$$

where T is global warming since pre-industrial times, S is cumulative carbon emissions, and the temperature adjusts at the rate ϵ which parameterises the pulse-adjustment timescale of the climate system (Allen et al., 2009; Dietz and Venmans, 2019). As in Matthews et al. (2009), Allen et al. (2009), van der Ploeg (2018) and Dietz and Venmans (2019), global warming is approximately linearly proportional to cumulative carbon emissions, and the time-invariant parameter ζ is the transient climate response to cumulative emissions (TCRE). The cumulative carbon emissions evolve according to the following law of motion:

$$\dot{S}(t) = E(t), \tag{7}$$

where E is the instantaneous flow of carbon emissions. The carbon-intensive and clean capital stocks are accumulated according to the following laws of motion:

$$\dot{K}_D(t) = I_D(t) - R(t) - \delta K_D(t), \tag{8a}$$

$$\dot{K}_C(t) = I_C(t) - \frac{\phi}{2}\frac{I_C(t)^2}{K_C(t)} + R(t) - \frac{\kappa}{2}\frac{R(t)^2}{K_C(t)} - \delta K_C(t), \tag{8b}$$

where $I_D(t)$ and $I_C(t)$ are the amount of investments in carbon-intensive and clean capital at time t, respectively (cf. Hambel et al., 2020). Clean capital investment is subject to intertemporal adjustment costs *à la* Hayashi (1982), where ϕ is the parameter of intertemporal investment adjustment cost. Capital goods can be reallocated between carbon-intensive and clean sectors at intersectoral reallocation costs. R is the amount of capital goods reallocated from dirty to clean sectors. Intersectoral reallocation of capital generates quadratic costs governed by the parameter κ. Intuitively, one unit of carbon-intensive capital can be reallocated into less than one unit of clean capital where the wedge increases in the amount being reallocated. δ is the rate of capital depreciation.

We describe the climate damages by a convex disutility function of warming, $V(T) := \frac{1}{2}\chi T^2$, where χ is the coefficient of marginal warming damages. The economy has a representative household (with a unit mass) with logarithmic utility in consumption, C, i.e., $U(C) := \ln(C)$. Preferences are additively separable over consumption C and warming T. Hence, we consider an optimal growth problem that maximizes

$$\max_{[C(t), I_D(t), R(t)]_{t=0}^{\infty}} \int_0^{\infty} e^{-\rho t} \left(\ln C(t) - \frac{1}{2}\chi T(t)^2 \right) dt, \tag{9}$$

subject to the law of motion for temperatures (6), cumulative carbon emissions (7), carbon-intensive capital (8a) and carbon-free capital (8b), and the aggregate resource constraint

$$Y(t) = C(t) + I_D(t) + I_C(t), \tag{10}$$

given the initial conditions $K_D(0) = K_D^0$, $K_C(0) = K_C^0$, $S(0) = S_0$ and $T(0) = T_0$.

2.2 Efficiency Conditions

The Maximum Principle of optimal control is used to solve the problem of maximizing (9) subject to (6), (7), and (8). The optimal growth path is characterized by the following efficiency conditions:

$$C = \lambda_D^{-1}, \tag{11a}$$

$$I_C = \phi^{-1}(1 - \lambda_D/\lambda_C) K_C, \tag{11b}$$

$$R = \kappa^{-1}(1 - \lambda_D/\lambda_C) K_C, \tag{11c}$$

$$(\rho + \delta)\lambda_D - \dot{\lambda}_D = \lambda_S P_{K_D} + U' F_{K_D}, \tag{11d}$$

$$(\rho + \delta)\lambda_C - \dot{\lambda}_C = \lambda_S P_{K_C} + U' \left(F_{K_C} + \frac{0.5\phi (I_C/K_C)^2 + 0.5\kappa (R/K_C)^2}{1 - \phi I_C/K_C} \right), \tag{11e}$$

$$\rho \lambda_S - \dot{\lambda}_S = \epsilon \zeta \lambda_T, \tag{11f}$$

$$(\rho + \epsilon)\lambda_T - \dot{\lambda}_T = -\chi T, \tag{11g}$$

and the transversality conditions: $\lim_{t\to\infty} e^{-\rho t}\lambda_D K_D = 0$, $\lim_{t\to\infty} e^{-\rho t}\lambda_C K_C = 0$, $\lim_{t\to\infty} e^{-\rho t}\lambda_S S = 0$, and $\lim_{t\to\infty} e^{-\rho t}\lambda_T T = 0$ where $\lambda_D > 0$, $\lambda_C > 0$, $\lambda_S < 0$ and $\lambda_T < 0$ are the shadow values corresponding to K_D, K_C, S and T, respectively. More specifically, $-\lambda_S/\lambda_D = -C\lambda_S > 0$ represents the social cost of carbon in units of final goods, and $\lambda_T < 0$ the welfare cost of marginal increases in temperature.

Condition (11b) show that the amounts of clean capital investment are large if the parameter of investment adjustment costs ϕ is small and if the relative ratio of shadow prices between dirty and clean capital λ_D/λ_C is low. Condition (11c) shows that the amounts of capital goods reallocated from dirty to clean sectors rise in proportion to the stock of clean capital K_C and decreases in the relative ratio of shadow prices between clean and dirty capital λ_D/λ_C. The rate of intersectoral capital reallocation is larger if the intersectoral reallocation cost parameter κ is smaller. Conditions (11d)-(11e) imply that the ratios of shadow values between carbon-intensive and clean capital (or the ratios of Tobin's Q) is given by

$$\frac{\lambda_D(t)}{\lambda_C(t)} = \frac{\int_t^\infty e^{-(\rho+\delta)(t'-t)}(\lambda_S P_{K_D} + U' F_{K_D})dt'}{\int_t^\infty e^{-(\rho+\delta)(t'-t)}\left(\lambda_S P_{K_C} + U'\left(F_{K_C} + \frac{0.5\phi(I_C/K_C)^2 + 0.5\kappa(R/K_C)^2}{1-\phi I_C/K_C}\right)\right)dt'}. \tag{12}$$

The efficient growth path has positive amounts of investment and intersectoral reallocation for accumulating clean capital if its shadow value is larger than the carbon-intensive one, $\lambda_C > \lambda_D$. Equation (12) shows that allocating resources towards investment in carbon-intensive capital incurs increasing marginal costs due to convex warming damages. In contrast, augmenting clean capital gains substantial benefits by offsetting carbon emissions and warming damages. Also an augmented stock of clean capital can reduce the costs associated with investment adjustment and intersectoral capital reallocation costs.

As in Dietz and Venmans (2019), integrating the law of motion for temperature given in (6) yields a closed-form expression of the temperature as follows:

$$T(t) = \int_{-\infty}^{t} \exp(-\epsilon(t-t'))\epsilon\zeta S(t')dt' \approx \frac{\epsilon}{\epsilon+\vartheta}\zeta S(t) \tag{13}$$

where ϵ is the rate of pulse-adjustment of the climate system and takes a central value of $\epsilon = 0.5$ as in Allen et al. (2009), Ricke and Caldeira (2014) and Dietz and Venmans (2019). Hence, the value of the integral (13) is dominated by the most recent few years. To determine global warming at time t it is sufficient to know cumulative emissions at the same time, and the history of emissions has little effect. In other words, the climate system responds very quickly to the emission impulse, and temperature inertia is weak. Over such a short period of climate adjustment, we can treat the growth of cumulative emissions as a constant, and at the intertemporal equilibrium the growth rate of cumulative emission is $\vartheta := \dot{S}/S = 0$ as in Dietz and Venmans (2019). Accordingly, the linear climate model dictates that temperature is given by

$$T(t) \approx \zeta S(t), \tag{14}$$

where the time-independent coefficient ζ is the TCRE, and warming is approximately linearly proportional to cumulative emissions of carbon as in Matthews et al. (2009), Allen et al. (2009), van der Ploeg (2018) and Dietz and Venmans (2019).

Furthermore, as in Dietz and Venmans (2019), the climate system adjusts quickly in response to emissions, so the marginal disutility of warming becomes constant after the short adjustment period. This allows us to make the following approximation of the marginal damages of warming λ_T given in (11g):

$$\lambda_T(t) = -\int_t^\infty \exp(-(\rho+\epsilon)(t-t'))\chi T(t')dt' \approx -\frac{\chi}{\rho+\epsilon}T(t) = -\frac{\chi\zeta}{\rho+\epsilon}S(t) < 0, \tag{15}$$

where the marginal disutility from the temperature perturbation is discounted by a delay-adjusted rate $\rho+\epsilon$. Given (15), the law of motion for the shadow cost of carbon given by (11f) can be rewritten as

$$\rho\lambda_S(t) - \dot{\lambda}_S(t) = \epsilon\zeta\lambda_T(t) = -\frac{\epsilon\zeta^2\chi}{\rho+\epsilon}S(t), \tag{16}$$

where χ is the coefficient of marginal warming damages, and ϵ governs the rate of climate adjustment as a response to an emission impulse.

We can show that the optimal allocations as characterized by (11) can be implemented in the market equilibrium by pricing carbon emissions at a level equal to the social cost of carbon, i.e., $\tau = -\lambda_S/U'(C) = -C\lambda_S > 0$, where the shadow cost of carbon in the utility units is converted to final goods units by dividing marginal utility of consumption (see Appendix A.1 for details).

3 Avoiding the Running Down of Carbon-Intensive Capital

In this section, we show analytically that 1) without accumulating clean capital to offset warming damages, a tightening of climate regulations leads to the running down of carbon-intensive capital, and 2) the stepping-up of clean capital accumulation curbs the running down of carbon-intensive capital caused by climate regulations.

For that purpose, we consider a problem that maximizes (9) subject to the law of motion for both carbon-intensive capital and cumulative emissions: $\dot{K}_D(t) = \eta K_D(t)^\alpha \bar{K}_C^{1-\alpha} - C(t) - \delta K_D(t)$ and $\dot{S}(t) = \xi K_D^\beta \bar{K}_C^{-\beta}$, and the temperature equation $T(t) = \zeta S(t)$, given their initial conditions $K_D(0) = K_D^0$ and $S(0) = S_0$. Economic growth is only driven by carbon-intensive capital without accumulating clean capital over time through investment and capital reallocation, i.e., $I_C(t)=0$ and $R(t)=0$, $\forall t\in[0,\infty)$. The clean capital stock thus remains unchanged at $K_C(t)=\bar{K}_C$, $\forall t\in[0,\infty)$. The functions of emissions and production are given by (4) and (5), respectively. Transitional dynamics of the growth path are characterized by

$$\frac{\dot{K}_D}{K_D}=\eta\left(\frac{K_D}{\bar{K}_C}\right)^{\alpha-1}-\frac{1}{\lambda_D K_D}-\delta, \qquad \dot{S}=\xi\left(\frac{K_D}{\bar{K}_C}\right)^{\beta}, \tag{17a}$$

$$\frac{\dot{\lambda}_D}{\lambda_D}=\rho+\delta-\eta\alpha\left(\frac{K_D}{\bar{K}_C}\right)^{\alpha-1}-\xi\beta\lambda_S\left(\frac{K_D}{\bar{K}_C}\right)^{\beta}\frac{1}{\lambda_D K_D}, \qquad \dot{\lambda}_S=\rho\lambda_S+\frac{\epsilon\zeta^2\chi}{\rho+\epsilon}S, \tag{17b}$$

where (17) describes the dynamics of carbon-intensive capital and cumulative emissions $[K_D,S]$ and their corresponding shadow values $[\lambda_D,\lambda_S]$.

In our analytical framework, a tightening of climate policy regulations is equivalent to correcting for a higher level of warming damages, i.e., $V(T)=\frac{1}{2}\chi T^2$. An increase in the coefficient of marginal warming damages, χ, thus corresponds to a tightening of climate regulations.[5] The following proposition gives the effect of tightening climate regulations on the stock of carbon-intensive capital.

Proposition 1. *Without accumulating clean capital to offset carbon emissions and global warming damages, a tightening of climate regulations leads to running down of carbon-intensive dirty capital.*

Proof. See Appendix A.2. □

Figure 2 provides the phase diagram to illustrate the result of Proposition 1. If climate regulations are tightened to correct for global warming damages caused by carbon-intensive dirty capital, the stable equilibrium path of carbon-intensive capital shifts downwards from the solid blue to the solid red line, and the transitional dynamics will end up with smaller capital stocks. This change leads to an outcome where a portion of carbon-intensive capital has to be ran down. Intuitively, without taking measures to offset climate damages caused by carbon-intensive capital, stringent regulations put a tighter constraint on the stock of carbon-intensive capital that is compatible with the environmental constraint. The portion of carbon-intensive capital outside the constraint needs to be displaced and written off. This result echoes existing findings that running down of carbon-intensive assets can be attributed to a tightening of climate regulations (e.g., van der Ploeg and Rezai, 2020a,b).

Furthermore, we can show that an efficient transition to green growth necessitates dynamic accumulation of clean capital over time, and the stepping-up of clean capital accumulation acts to lessen the running down of carbon-intensive capital. The following proposition gives the result.

Proposition 2. *The efficient growth path is characterized by a positive amount of capital investment and intersectoral reallocation to augment the clean capital stock. An increase in the stock of clean capital*

[5] Allocations in a market equilibrium with climate policy regulations in the form of pricing carbon according to marginal warming damages $\tau=\frac{-\lambda_S}{U'}$ are consistent with the optimum internalizing climate damages caused by carbon emissions.

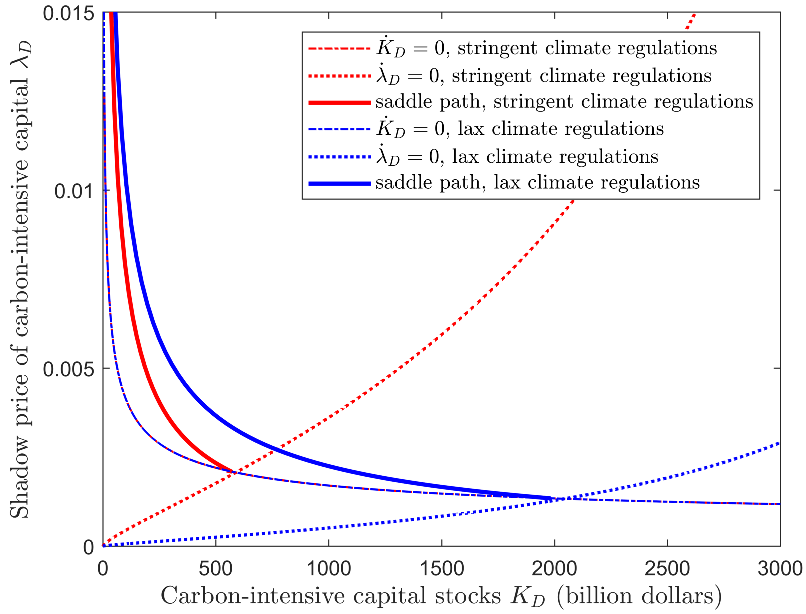


Figure 2: Numerical Illustration of Proposition 1. The phase diagram of transitional dynamics is plotted with different levels of stringency of climate regulations. The red line corresponds to stringent climate regulations with a larger coefficient of marginal warming damages χ. The blue line denotes lax climate regulations with a smaller coefficient of χ. The dashed and dotted lines depict the locus of points $\dot{K}_D=0$ and $\dot{\lambda}_D=0$, respectively. The solid line depicts the stable saddle path of transitional dynamics. When climate regulations become tighter, the equilibrium path of carbon-intensive capital shifts from the solid blue to the solid red line, and the economy builds up much less carbon-intensive capital. A portion of carbon-intensive capital (the gap between blue and red lines) thus has to be ran down. The phase diagram is produced based on numerical simulations of (17), with the parameter inputs $\eta=2.14$, $\alpha=0.42$, $\beta=0.58$, $\rho=0.05$, $\delta=0.06$, $\xi=0.15$, $\epsilon=0.5$, $\zeta=0.002$, $\bar{K}_C=128$, $\chi=0.012$ (lax climate regulations) and $\chi=0.12$ (stringent climate regulation).

offsets global warming damages and lessens the running down of carbon-intensive capital caused by climate regulations.

Proof. See Appendix A.3. □

Intuitively, since an augmented stock of clean capital generates substantial dynamic benefits (as measured by shadow values) by offsetting warming damages, it is dynamically efficient to allocate resources towards augmenting clean capital through both investment and intersectoral capital reallocation. Then the augmented stock of clean capital has an effect on the running down of carbon-intensive capital. As Figure 3 illustrates, an increase in the stock of clean capital changes the transitional dynamic path of carbon-intensive capital, leading to a shift from the solid red to the solid black line. With a larger stock of clean capital, the carbon-intensive capital stock tends to be larger, and the degree of running

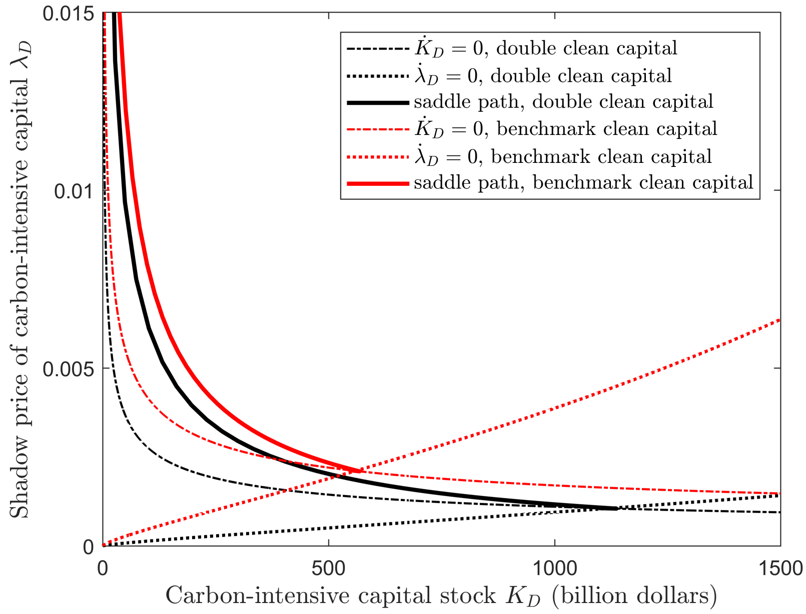


Figure 3: Numerical Illustration of Proposition 2. The phase diagram of transitional dynamics is plotted with different stocks of clean capital. The red line plots the growth path of carbon-intensive capital with a smaller clean capital stock $\bar{K}_C$, and the black line corresponds to the case with a larger clean capital stock $\bar{K}_C$. The dashed and dotted lines depict the locus of points $\dot{K}_D = 0$ and $\dot{\lambda}_D = 0$, respectively. The solid lines depict the stable saddle path of transitional dynamics. With a larger stock of clean capital, the equilibrium path of carbon-intensive capital shifts from the solid red to the solid black line, and the economy can build up a larger stock of carbon-intensive capital. Hence, the degree of running down carbon-intensive capital caused by stringent climate regulations becomes less severe when the clean capital is augmented. The phase diagram is produced based on numerical simulations of (17), with the parameter inputs $\eta = 2.14$, $\alpha = 0.42$, $\beta = 0.58$, $\rho = 0.05$, $\delta = 0.06$, $\xi = 0.15$, $\epsilon = 0.5$, $\zeta = 0.002$, $\chi = 0.12$, $\bar{K}_C = 128$ (benchmark clean capital) and $\bar{K}_C = 256$ (double clean capital).

down carbon-intensive capital caused by climate regulations becomes less severe.

We show above how the running down of carbon-intensive assets could be potentially minimized in the transition to green growth. The point is that the interplay between carbon-intensive and carbon-free capital could benefit each other. On the one hand, stepping-up of clean capital accumulation as induced by stringent climate regulations offsets global warming damages, protects the economic value of carbon-intensive capital, and lessens the running down of carbon-intensive capital. On the other hand, not running down carbon-intensive capital provides more economic means to facilitate clean capital accumulation through the channels of investment and intersectoral capital reallocation. Through this mechanism, both carbon-intensive capital (using fossil energy) and carbon-free clean capital (using renewable energy, high-efficiency, low-emission technologies, and pollution control facilities, etc.) tend to coexist. Clean capital enables the use of carbon-intensive capital, which otherwise would have to be run down. Hence, accumulating carbon-free clean capital to offset emissions and global warming damages caused by carbon-intensive capital provides a potential mechanism for avoiding the running down of carbon-intensive assets in the transition to green growth.

4 Green Endogenous Growth

The interaction between carbon-intensive and clean capital generates a relationship where each type of capital provides benefits to the other. Clean capital offsets warming damages and protects the economic values of carbon-intensive capital. Carbon-intensive capital provides economic means to facilitate clean capital accumulation via investment and intersectoral capital reallocation. As a result, both carbon-intensive and carbon-free capital can mutually benefit each other and are needed in the transition to green growth.

We show that the transition to green growth can create an endogenous growth path with sustained accumulation of both carbon-intensive and clean capital. To characterize the endogenous growth path, we define

$$g := \frac{\dot{K}_D}{K_D}, \qquad k := \frac{K_D}{K_C}, \qquad c := \frac{C}{K_D}, \tag{18}$$

where g is the rate of dirty capital accumulation, k the ratio of carbon-intensive to carbon-free capital, and c the ratio of consumption to carbon-intensive capital. The growth rates of clean capital and consumption are defined, respectively, by

$$\frac{\dot{K}_C}{K_C} = g - \frac{\dot{k}}{k}, \qquad \frac{\dot{C}}{C} = g + \frac{\dot{c}}{c}. \tag{19}$$

The following proposition characterizes the green endogenous growth.

Proposition 3. *The equilibrium path of green endogenous growth as determined by* $[g,k,c,S,T,\lambda_S,\lambda_T]$ *is characterized by*

i. the law of motion for the ratio of carbon-intensive to clean capital, k:

$$\frac{\dot{k}}{k} = \frac{\phi\kappa}{2(\phi+\kappa)}(\eta k^{\alpha} - (c+g+\delta)k)^2 - (\eta k^{\alpha} - (c+g+\delta)k) + \delta + g; \tag{20}$$

ii. the law of motion for the ratio of consumption to carbon-intensive capital, c:

$$\frac{\dot{c}}{c} = \eta\alpha k^{\alpha-1} + \xi\beta\lambda_S c k^{\beta} - \rho - \delta - g; \tag{21}$$

iii. the efficiency condition for carbon-intensive and clean capital investments:

$$\eta\alpha k^{\alpha-1}+\xi\beta\lambda_S ck^{\beta}=\left(\eta(1-\alpha)k^{\alpha}-\xi\beta\lambda_S ck^{\beta+1}\right)\left(1-\frac{\phi\kappa}{\kappa+\phi}(\eta k^{\alpha}-(c+g+\delta)k)\right)+\frac{\phi\kappa}{2(\kappa+\phi)}(\eta k^{\alpha}-(c+g+\delta)k)^2; \tag{22}$$

iv. the law of motion for cumulative emissions of carbon, S:

$$\dot{S}=\xi k^{\beta}; \tag{23}$$

v. the law of motion for the shadow value of cumulative carbon emissions, λ_S:

$$\dot{\lambda}_S=\rho\lambda_S-\epsilon\zeta\lambda_T; \tag{24}$$

vi. the law of motion for change in temperature, T:

$$\dot{T}=\epsilon(\zeta S-T); \tag{25}$$

vii. the law of motion for the shadow value of global warming, λ_T:

$$\dot{\lambda}_T=(\rho+\epsilon)\lambda_T+\chi T. \tag{26}$$

Proof. See Appendix A.4. □

The characterization of green endogenous growth builds on the efficient growth allocations characterized by (11). First, (20) incorporates the law of motion for carbon-intensive and clean capital given in (8). Second, (21) provides the Euler consumption rule, which combines the efficiency conditions (11a) and (11d). Third, (22) gives the efficiency condition for carbon-intensive and clean capital investments from (11b), (11d) and (11e). Capital investment generates effects on the economy through the channels of both production and global warming damages. Investments in dirty and clean capital both increase outputs through the production channel. Investing in carbon-intensive capital accelerates carbon emission and warming damages, while accumulating carbon-free clean capital acts to offset emissions and mitigate warming damages.

We calibrate the model and simulate the transitional dynamics of green endogenous growth. Table I provides the benchmark calibration of the model parameters. The rate of time preference is set at a level $\rho=0.05$ which is consistent with the standard range. The global warming damages caused by cumulative emissions of carbon are described by a convex quadratic function. The coefficient of marginal disutility of warming is set to $\chi=0.012$ as in van der Ploeg and Withagen (2012, 2014). The installed capacities of

Table I: Parameter Values

Parameters	Symbol	Value
Cobb-Douglas weight	θ	0.65
productivity loss from emissions	γ	0.4
output elasticity of carbon-intensive capital	$\alpha := \theta - \frac{1-\theta}{1-\gamma}\gamma$	0.42
emission elasticity of carbon-intensive capital	$\beta := \frac{1-\theta}{1-\gamma}$	0.58
carbon intensity-output relationship	ψ	0.32
emission intensity parameter	$\xi := \psi^{\frac{1}{1-\gamma}}$	0.15
productivity of capital	$\eta := \psi^{\frac{-\gamma}{1-\gamma}}$	2.14
investment adjustment cost parameter	ϕ	18
intersectoral capital reallocation cost parameter	κ	5
rate of time preference	ρ	0.05 (annual)
rate of capital depreciation	δ	0.06 (annual)
rate of climate adjustment	ϵ	0.5 (annual)
transient climate response to cumulative emissions	ζ	2 (°C/TtC)
marginal disutility of gobal warming	χ	0.012
initial condition of dirty-clean capital ratio	$k(0)$	50
initial condition of cumulative carbon emissions	$S(0)$	400 (GtC)
initial condition of temperature increase	$T(0)$	1 (°C)

fossil- and renewable-based power generation are used as a proxy for carbon-intensive and clean capital, respectively. Based on the data of China's electric power industry over the period 1949-2016 (China Electric Power Press, 2017a,b), the fossil-fired power plants account for approximately 65% in total power generation, and we thus calculate the Cobb-Douglas weight of production technology as $\theta = 0.65$. The productivity loss from pollution emissions is determined by the parameter γ, which is set to $\gamma = 0.4$ according to Fu et al. (2018) and He et al. (2019).[6] Accordingly, we set the parameter of output elasticity of carbon-intensive capital to $\alpha := \theta - \frac{1-\theta}{1-\gamma}\gamma = 0.42$, and the parameter of emission elasticity to $\beta := \frac{1-\theta}{1-\gamma} = 0.58$. Furthermore, following Hambel et al. (2020), we set the parameters of investment adjustment costs and intersectoral capital reallocation costs to $\phi = 18$ and $\kappa = 5$. We collect the data of China's carbon emission intensity (CO_2 per unit of GDP) and GDP over the period 1970-2016 from Global Carbon Project (2019) and Feenstra

[6]Empirical studies such as Graff Zivin and Neidell (2012), Chang et al. (2016), Fu et al. (2018), Chang et al. (2019) and He et al. (2019) uncover statistically significant adverse effects of pollution on productivity, with an estimated elasticity of 0.26 in agriculture (Graff Zivin and Neidell, 2012), 0.31-0.44 in manufacturing sectors (Fu et al., 2018; He et al., 2019), and 0.05-0.35 in service sectors (Chang et al., 2016, 2019). We consider electricity and power generation as a manufacturing sector and set the elasticity parameter $\gamma = 0.4$ to describe the effect of pollution on productivity.

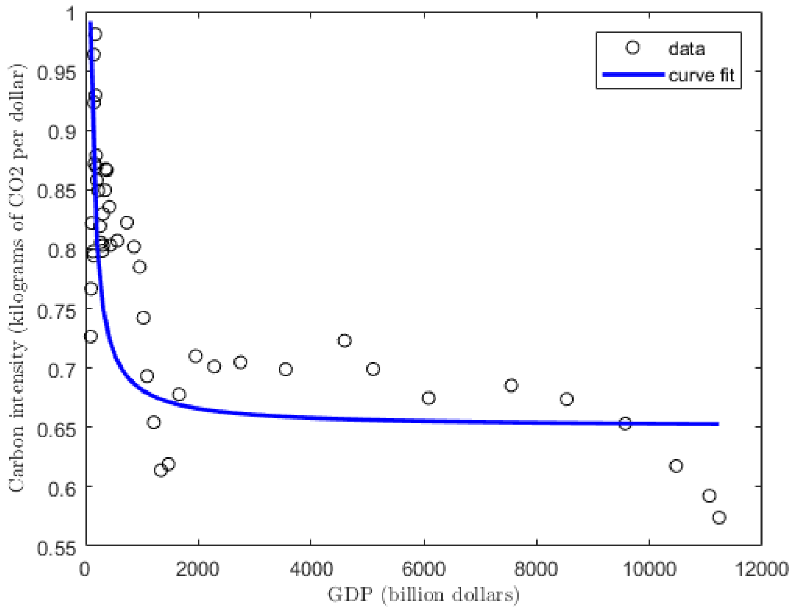


Figure 4: The Decline of the Carbon Emission Intensity. This figure plots the data of China's carbon emission intensity (vertical axis) versus GDP (horizontal axis) over the period 1970-2016. Carbon intensity is measured in kilograms of CO2 per dollar, and GDP is measured in billions dollars. The curve fit is produced using the inverse proportional function (3), $m(t)=\psi Y(t)^{-1}$, where $m(t)$ and $Y(t)$ are carbon emission intensity and GDP at time t, respectively. The curve fitting yields the calibrated parameter $\psi=0.3162$.

et al. (2015), and fit their relation using the inverse proportional function (3). The parameter of carbon intensity-output relationship is estimated as $\psi=0.32$. Then we calculate the coefficient of carbon emission equation as $\xi:=\psi^{\frac{1}{1-\gamma}}=0.15$ and the coefficient of capital productivity as $\eta:=\psi^{\frac{-\gamma}{1-\gamma}}=2.14$ (see Figure 4).

Following Ricke and Caldeira (2014) and Dietz and Venmans (2019), the rate of climate adjustment is set to $\epsilon=0.5$. Following Matthews et al. (2009), Allen et al. (2009), van der Ploeg (2018) and Dietz and Venmans (2019), we calculate the transient climate response to cumulative emissions (TCRE) as 0.54°C per trillion tons of CO2, or equivalently, 1.98°C per trillion tons of carbon. The TCRE is thus set to $\zeta=2$ °C/TtC. Finally, the initial year is set to 2015, and the time period of simulation is 200 years. According to BP (2020), renewable-based energy (solar, wind, biofuels, and other renewables) are approximately 1.7% of total energy production (2564 TWh), and fossil fuels (coal, oil and gas) account for roughly 86.5% (129517 TWh) in 2015. The initial value of the ratio of carbon-intensive to clean capital is thus set to $k(0)=86.5/1.7\approx 50$. van der Ploeg (2018) and Dietz and Venmans (2019) suggest that the initial 2015 cumulative carbon emissions and temperature anomaly relative to pre-industrial times are roughly 400 GtC and 1 degrees Celsius, respectively. We thus set the initial value of cumulative carbon emissions and temperature increase to $S(0)=400$ and $T(0)=1$, respectively.

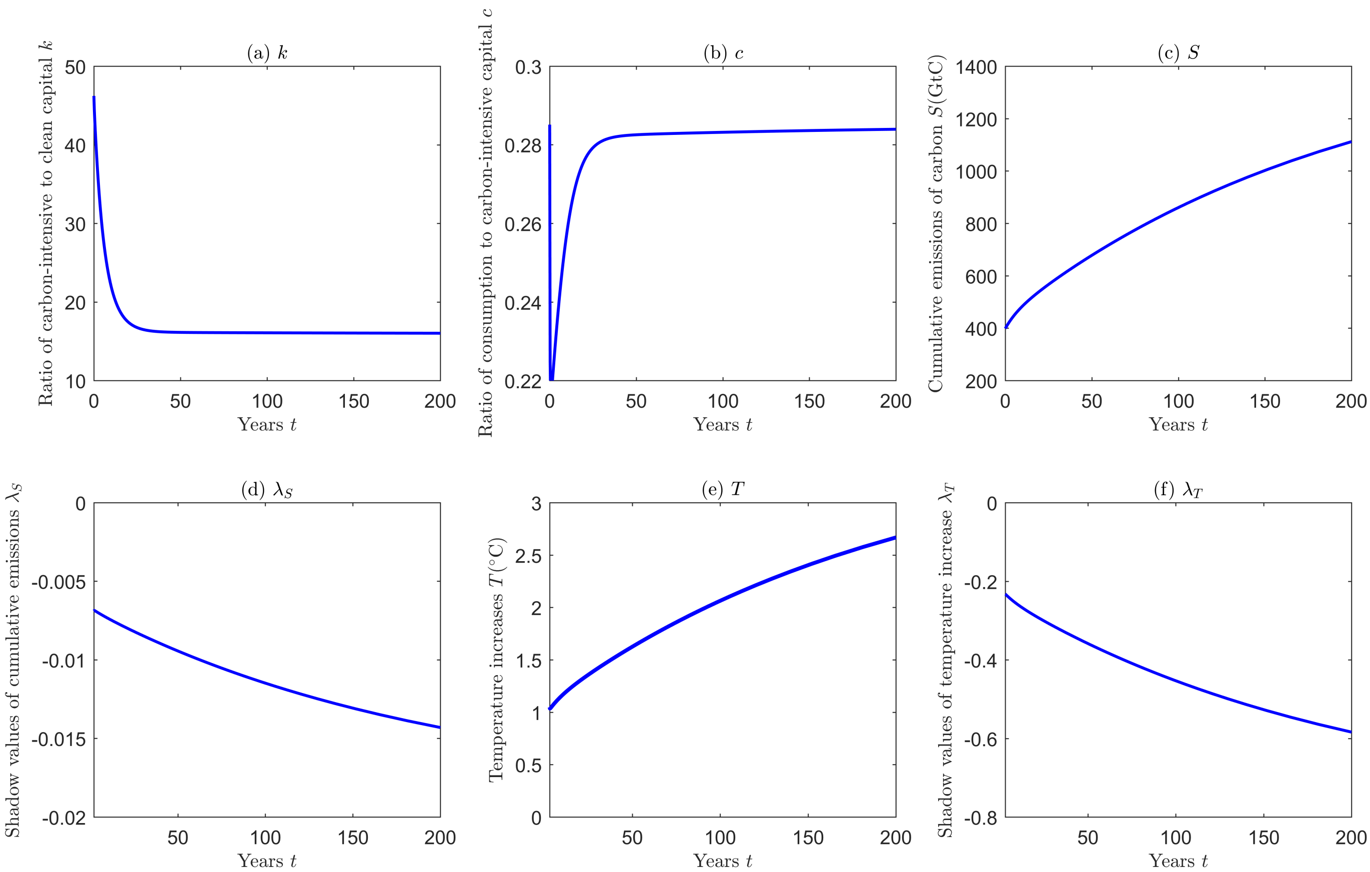


Figure 5: The Time Path of $[k,c,S,\lambda_S,T,\lambda_T]$ in Green Endogenous Growth. Panel (a) plots a decreasing trend of the ratio of carbon-intensive to clean capital $k := K_D/K_C$. Panel (b) simulates an increasing time path of the ratio of consumption to carbon-intensive capital $c := C/K_D$. Panel (c) depicts the time path of cumulative carbon emissions S that is increasing at a positive and diminishing rate. Panel (d) shows the time path of the shadow value of cumulative emissions λ_S. Panel (e) plots that the degree of temperature anomaly relative to pre-industrial times is increasing over time. Panel (f) shows that the shadow value of warming is negative and increasing. The time period of simulation is 200 years.

Based on the parameter inputs in Table I, we simulate the time path of green endogenous growth characterized by (20)-(26).[7] Figure 5 shows the time path of $[k,c,S,T,\lambda_S,\lambda_T]$ along the green endogenous growth path. Panel (a) plots the time path of the ratio of carbon-intensive to clean capital $k := K_D/K_C$. It features a gradual decline over time after an initial quick adjustment in response to emission impulses. This changing trend in $k := K_D/K_C$ suggests that clean capital K_C needs to gain a bigger share in the capital portfolio for generating endogenous growth subject to warming damages. Panel (b) depicts the time path of the ratio of consumption to carbon-intensive capital $c := C/K_D$ which features a gradual increase over the time horizon. This trend suggests that consumption relative to carbon-intensive capital, after an

[7] Appendix A.5 provides the details of numerical implementation procedures.

initial instantaneous downward jump, increases over time along the endogenous growth path. In panel (c), the stock of cumulative carbon emissions S accumulates over time from the initial value of 400 GtC to 1000-1200 GtC after 200 years. Since clean capital accumulation is stepped up to offset emission growth, the curve of cumulative emissions tends to flatten as a larger stock of clean capital is accumulated. In panel (d), the shadow value of cumulative emissions is negative and decreasing, because the impulse of carbon emission leads to temperature increases and incurs warming damages on the economy. In panel (e), the degree of temperature anomaly relative to pre-industrial times increases from the initial 1°C to $2.5-3$°C after 200 years. Temperature increases are approximately linear in cumulative carbon emissions. The climate system adjusts quickly in response to emissions, and the temperature thus rises with cumulative emissions. In panel (f), the shadow value of global warming is negative and decreasing, implying that the welfare cost of warming increases as temperature rises and a higher level of warming damages is incurred.

Figure 6 simulates the time path of aggregate variables during the phase of transitional dynamics. Panel (a)-(b) shows that the efficient path of green endogenous growth generates a shadow value of clean capital that is much larger than the carbon-intensive one. Clean capital can create larger dynamic benefits by offsetting carbon emissions and warming damages. It is thus efficient to step up clean capital accumulation through the channels of both investments and reallocations of capital goods from carbon-intensive to clean sectors. Panel (c) simulates the growth rates of capital, consumption, and production outputs. It is shown that the growth rates differ during the phase of transitional dynamics, but the difference tends to be more limited over time, with a rapid convergence to the same growth rate. Since achieving green endogenous growth needs to build up more clean capital to offset warming damages caused by carbon-intensive dirty capital, the green endogenous growth path features a larger rate of clean capital accumulation. Panel (d) shows the sustained accumulation of both carbon-intensive capital and clean capital (with a log scale) along the endogenous growth path though it is subject to convex warming damages. This result thus suggests that both carbon-intensive capital and carbon-free capital can potentially coexist in the transition to green growth. Stepping-up of carbon-free clean capital, by offsetting carbon emissions and global warming damages, plays a crucial role to avoid premature running down of carbon-intensive capital in the transition to green endogenous growth. Panel (e) plots the endogenous growth paths of consumption, capital investment, and intersectoral capital reallocation (with a log scale). They all show sustained growth along the endogenous growth path. Since the accumulation of clean capital generates a larger shadow value (cf. panel (a)), growing alone the endogenous growth path will induce positive amounts of both investment and intersectoral reallocation of capital goods from carbon-intensive to clean sectors. Meanwhile, capital investment incurs higher adjustment costs than intersectoral capital reallocation, i.e., $\phi > \kappa$, as in Hambel et al. (2020). The simulations indicate that the amount of clean capital investment is relatively smaller than that of intersectoral capital reallocation. Panel (f) plots the time path of the social cost of carbon

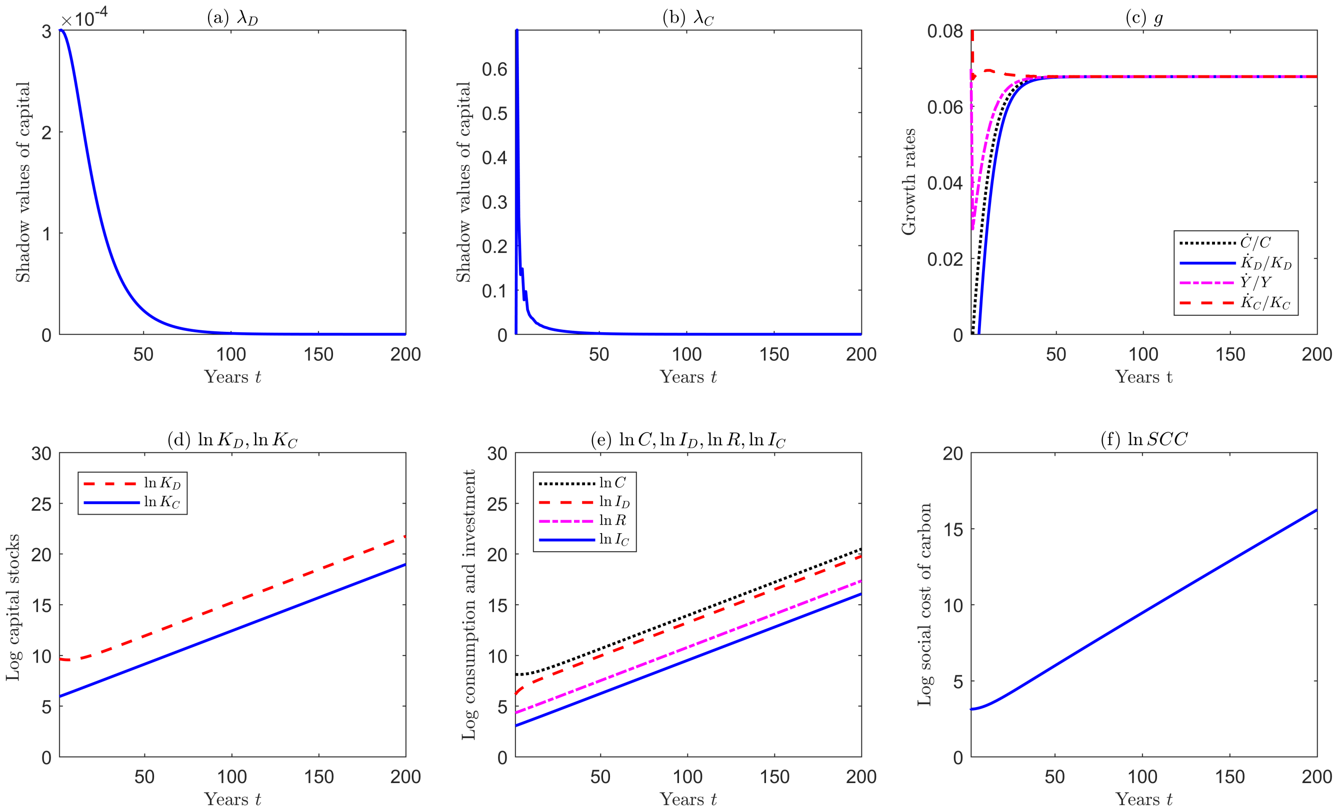


Figure 6: The Time Path of $[\lambda_D,\lambda_C,K_D,K_C,C,I_C,I_D,R,g,SCC]$ in Green Endogenous Growth. Panel (a) plots the time path of the shadow values of carbon-intensive capital λ_D. Panel (b) plots the time path of the shadow values of clean capital λ_C. The path of green endogenous growth determines a larger shadow value of clean capital as compared to carbon-intensive capital. Panel (c) shows the time path of the growth rates of consumption, capital and outputs. The growth rate of clean capital tends to be relatively larger as investments in clean capital are needed to offset warming damages caused by carbon-intensive capital. Panel (d) simulates the time path of carbon-intensive and clean capital stocks $[K_D,K_C]$ using the logarithmic scale. The green endogenous growth features the simultaneous accumulation of both carbon-intensive and clean capital. Panel (e) depicts the endogenous growth paths of consumption, investment, and capital reallocation $[C,I_C,I_D,R]$ using the logarithmic scale. Panel (f) simulates the time path of the social cost of carbon, SCC, using the logarithmic scale.

defined as the shadow value of cumulative emissions divided by the marginal utility of consumption. Due to the convex warming damages, the social cost of carbon rises over time at an increasing rate.

Figure 7 shows the time paths of flow and cumulative emissions of carbon along the endogenous growth path. As shown in panel (a), if climate regulations induce the build-up of clean capital to offset carbon emissions and global warming damages, the level of emission flows declines and becomes stationary. As shown in panel (b), the stabilized emission flows flatten the growth trend of cumulative emissions,

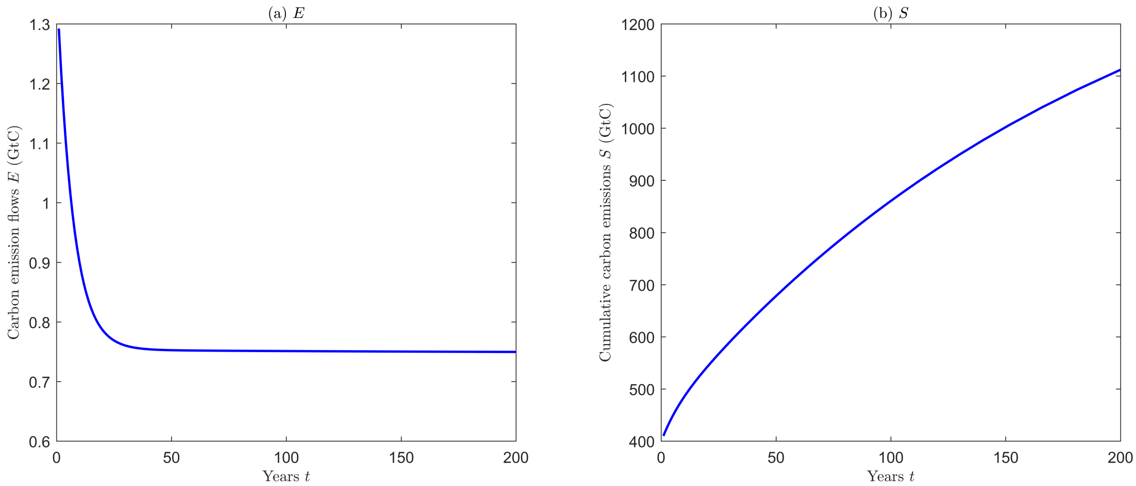


Figure 7: The Time Path of $[E,S]$ **in Green Endogenous Growth**. Panel (a) plots the time path of the emission flow of carbon, $E = \xi K_D^{\beta} K_C^{-\beta}$. The level of emission flows quickly becomes stationary when climate regulations induce stepping-up of clean capital to offset warming damages. Panel (b) simulates the rising time path of cumulative emissions of carbon, S.

thus facilitating a transition to green endogenous growth. Accordingly, along this endogenous growth path, both carbon-intensive and carbon-free capital is accumulated, consumption grows, and the curve of cumulative emissions is flattened. Stepping-up of clean capital accumulation, through offsetting emission and global warming damages, plays a crucial role to avoid the running down of carbon-intensive capital and generate an outcome of green endogenous growth. In contrast, without accumulating clean capital to offset carbon emissions and warming damages, climate regulations necessarily lead to the running down of carbon-intensive dirty capital.

5 Empirical Evidence

We have theoretically shown above that in the presence of environmental regulations, clean capital plays a positive role in avoiding the running down of carbon-intensive capital, and vice versa. The real-world stylized facts given in Section 1 also indicate the simultaneous accumulation of both types of capital without precipitous drops in carbon-intensive capital upon the arrival of clean ones. In this section, to further study the relationship between the two types of capital and provide empirical support for our analytical findings, we examine the interplay of the two types of capital stocks associated with carbon dioxide emissions. First, climate scientists have shown that the accumulation of greenhouse gases drives global warming, which has incurred environmental damages and utility losses. Our analysis in Section 4 has incorporated the stock of cumulative emissions as the main source of pollution damages. Second, we allow for emission flow pollutants such as sulfur dioxide, ozone, and fine particles which have been largely controlled by current local or regional policies. We construct a unique dataset for 27 countries by matching the OECD database with the World Bank database to verify the impact of clean capital on avoiding the running down of carbon-intensive capital stocks and assets. Carbon-intensive assets in this section refer to the capital stocks that produce carbon dioxide emissions. We start with a brief introduction of the data sources, then explain the empirical model and the econometric specification, and then provide robust evidence for the interplay between carbon-intensive and clean capital.

5.1 Data Sources

The OECD database. The data for OECD countries is provided by the OECD database. The estimation of capital stocks by country is essential for our empirical analysis. This database includes the net capital stock for most of its member countries. Also, the database provides the environmental stringency index, which is a key factor affecting the evolvement of capital growth. We use the environmental stringency index from the database as the proxy for environmental policy at the country level.

In the climate change context, our focus is on the stock pollutants such as carbon dioxide rather than emission flow pollutants such as fine particles. As most of the carbon dioxide emissions are derived from electricity generation, we rely on the data from electricity generation for estimating the composition of the capital stock. Specifically, we define the share of carbon-intensive capital in the total capital stock of a country as the share of electricity generated from fossil-fuel-based energy sources (coal, oil, and natural gas), while the share of clean capital is measured by the share of electricity generated using renewable technologies (e.g., hydro-power, solar energy, wind, nuclear energy).

The World Bank database. The World Development Indicators published by the World Bank provide information on the composition of electricity generation of its member countries. We obtain the share of fossil-fuel-based and clean electricity generation from the database to estimate the level of clean and carbon-intensive capital. For consistency, we use per capita carbon emissions from this database as the proxy for pollution stock. We then match data obtained from the two databases by the country identity. In total, we obtained values for 27 countries over the period 1996-2015. The final dataset for this analysis is an unbalanced panel due to missing values from the two databases. Table II displays descriptive statistics of the key variables.

To examine the relationship between clean and carbon-intensive capital empirically, we first employ the Fisher-type panel unit root test to show the stationary of each variable as it does not require strongly balanced data (Choi, 2001). The test shows that the unit root problem is shown at the level but not in the first difference among all variables. As the stationary assumption is required, we thus cannot use the

Table II: Descriptive Statistics of Key Variables

Variables	Mean	Std.Dev.	Min	Max
Original data:				
capital stock (index)[a]	85.58	14.58	26.46	109.97
share of clean capital (%)	29.93	24.73	0.58	99.72
environmental policy stringency (index)[b]	2.16	0.857	0.46	4.13
per capita emissions (metric tons per capita)	9.78	3.774	4.38	20.18
Estimated data:				
fossil-fuel-based capital stock (index)[a]	20.27	20.03	0.47	91.29
clean capital stock (index)[a]	65.31	23.86	0.20	96.56

[a] Data on capital stock from the OECD.Stat database are given in index with the year 2015 normalized to 100. The data for two types of capital stock is estimated based on the capital stock index and the values of clean capital shares accordingly.

[b] The OECD Environmental Policy Stringency Index is a country-specific and internationally -comparable measure of the stringency of environmental policy. It ranges from 0 (not stringent) to 6 (highest degree of stringency).

level variables for our analysis. In the following, we thus use first differences of variables for our empirical investigation.

5.2 Panel Vector Auto-regression (VAR) Model

We attempt to explore the dynamic interaction between clean and fossil-fuel-based capital using panel vector auto-regression (VAR), since it provides a strategy for clear identification of interactive variables. It offers useful forecasts of the magnitude and duration of bilateral interactions (Sims, 1980). As one of the powerful tools in macroeconomic literature, it has been used increasingly in the area of finance and political economy (e.g., Hasbrouck, 1991; Enders and Sandler, 1993). We follow Holtz-Eakin et al. (1988) and Canova and Ciccarelli (2009) to estimate the VAR model in country panel data. The panel VAR specification with order p is given as follows:

$$\begin{bmatrix} Ddirty_{i,t} \\ Dclean_{i,t} \end{bmatrix} = \sum_{p}^{j=1} \mathbf{A}_j \begin{bmatrix} Ddirty_{i,t-j} \\ Dclean_{i,t-j} \end{bmatrix} + \mathbf{B} \cdot \mathbf{X}_{i,t} + \mathbf{C}_i + \mathbf{D}_t + \boldsymbol{\epsilon}_{i,t}, \tag{27}$$

where $[Ddirty_{i,t} \quad Dclean_{i,t}]^\intercal$ is a 2×1 vector of dependent variables that includes the first difference of fossil-fuel-based capital, or say dirty capital growth $Ddirty_{i,t}$ and clean capital growth $Dclean_{i,t}$ in country i and at time t. $\mathbf{X}_{i,t}$ is a 2×2 matrix of exogenous control variables including environmental policy stringency and emissions per capita. $\mathbf{A}_j$ is the 2×2 matrix that captures the coefficients for the j-period lagged variables, $\mathbf{B}$ a 2×2 matrix of coefficients for $\mathbf{X}_{i,t}$, $\mathbf{C}_i$ the 2×1 vector of country fixed effects, $\mathbf{D}_t$ the 2×1 vector of time fixed effects, and $\boldsymbol{\epsilon}_{i,t}$ the 2×1 vector of serially uncorrelated error terms.

5.3 Empirical Results

We follow the overall coefficient of determination criterion to set the optimal lag order as $p=2$. In the following, the estimation of panel VAR with a lag order of $p=2$ is presented. Table III reports the results. Columns (1) and (2) present the results without controlling the environmental policy stringency and pollution stock, while columns (3) and (4) show the results with policy stringency and pollution stock as control variables. All four models control for country fixed effects and time fixed effects.

Table III: Empirical Results of Panel VAR Model

	Without control		**With control**	
	(1)	(2)	(3)	(4)
	Ddirty	*Dclean*	*Ddirty*	*Dclean*
L1.Ddirty	0.978	0.099	0.637	0.313
	(1.288)	(1.384)	(0.401)	(0.476)
L2.Ddirty	-0.026	0.209	-0.065	0.147
	(0.398)	(0.409)	(0.267)	(0.276)
L1.Dclean	1.287	-0.216	1.077**	-0.157
	(1.304)	(1.411)	(0.430)	(0.503)
L2.Dclean	0.242	-0.055	0.310	-0.234
	(0.430)	(0.444)	(0.254)	(0.268)
Policy stringency			-1.439*	1.550*
			(0.806)	(0.912)
Pollution stocks			-2.077**	2.490***
			(0.854)	(0.940))
Time fixed effects	Yes	Yes	Yes	Yes
Country fixed effects	Yes	Yes	Yes	Yes
Observations	292	292	284	284
Hansen's J test	$\chi^2 = 5.137$ (p=0.274)		$\chi^2 = 0.679$ (p=0.954)	
Granger causality test				
	χ^2	p value	χ^2	p value
Dclean → *Ddirty*	1.018	0.601	6.438	0.040
Ddirty → *Dclean*	0.393	0.822	0.546	0.761

Note: Standard error in parenthesis. Symbol "*L*." is the lag operator. *Ddirty* and *Dclean* are the growth of dirty and clean capital stocks, respectively. Significant levels are denoted as * of 10%, ** of 5%, and *** of 1%.

We find that there is no significant relationship between carbon-intensive capital growth and clean capital growth if the factor of policy stringency is missing. By adding this variable into our empirical specifications, we find that clean capital growth increases carbon-intensive capital growth, and this effect is statistically significant at the 5% level (see column (3)). Stringent environmental regulations reduce carbon-intensive capital growth and spur clean capital growth. As clean capital stimulates

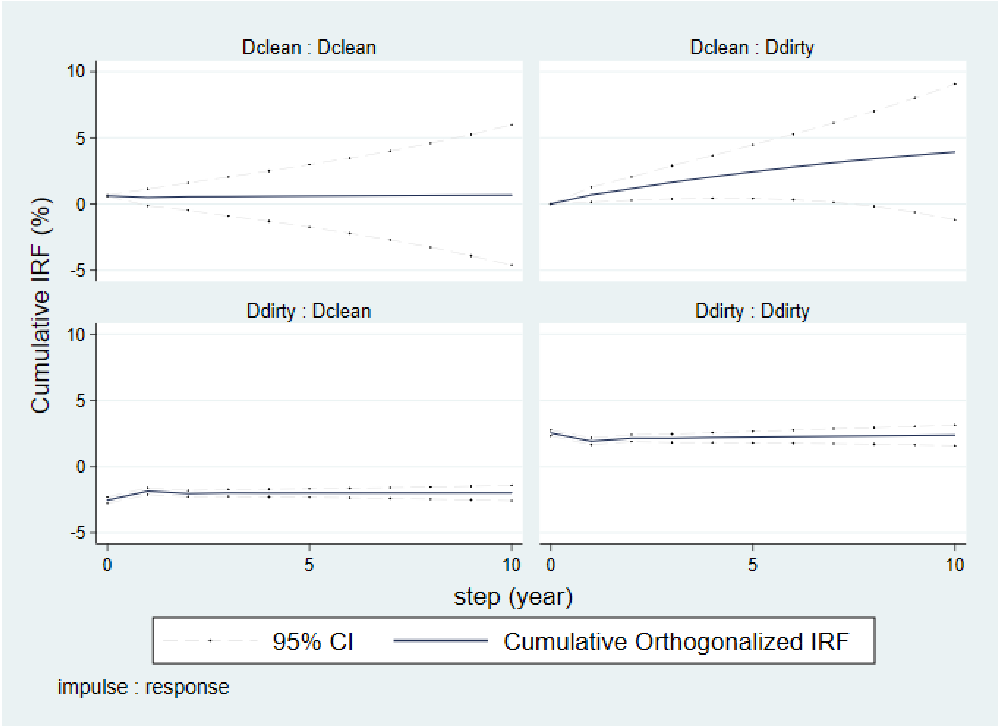


Figure 8: Cumulative Orthogonalized Impulse Response Function (IRF). Solid lines show the response of *Dclean* (left panels) and *Ddirty* (right panels) to a one-percent shock in *Dclean* (top panels) and *Ddirty* (bottom panels), respectively. *Dclean* and *Ddirty* refer to clean and dirty capital stock growth, respectively. The dashed lines represent 95% confidence interval (CI). The X-axis is the number of years after the shock, and the Y-axis is the cumulative orthogonalized impulse response function in percentage.

fossil-fuel-based capital growth, stringent environmental regulation indirectly accelerates fossil-fuel-based capital accumulation by boosting the growth of clean capital. This positive indirect effect mitigates the direct effect of environmental regulations on reducing the stock of carbon-intensive capital. Overall, the effectiveness of environmental regulation could thus be diluted.

Similarly, we find a significant effect of pollution stocks on the growth of both types of capital. It lowers dirty capital growth and pushes up clean capital growth. The accumulation of the pollution stock is also affected via the channel of clean capital. Table III also shows the results of the Granger causality tests after estimating the panel VAR model. It indicates that the null hypothesis that clean capital growth does not cause the change in dirty capital growth is rejected. That is, an increase in clean capital growth Granger causes an increase in carbon-intensive capital growth. Our results do not reject the opposite causality hypothesis. More importantly, we do not observe the causality relationship in the models without controlling the policy stringency and pollutant stock. Therefore, the empirical results are consistent with our theoretical findings.

To better explore the dynamic interaction between fossil-fuel-based and clean capital growth, we calculate the impulse response function (IRF) based on the panel VAR with control variables. Figure 8 shows the cumulative orthogonalized IRF to a one-percent shock in *Ddirty* and *Dclean*, respectively over 10 years. In response to a one-percent shock in *Dclean*, the IRF of *Dclean* goes up with a magnitude of 0.72%, and it stays persistently strong at around 0.5% level in the next ten years. Therefore, we observe a continuous increase over time from the top right panel of Figure 8, with a cumulative response of over 5% in ten years. There is no such significant and persistent response to the shocks for other panels.

Our empirical findings are consistent with our theoretical results. Stringent environmental regulations correcting for pollution damages tend to displace carbon-intensive capital and result in the running down of dirty capital. However, stepping-up clean capital as induced by stringent environmental regulations acts to decarbonize carbon-intensive capital and offset pollution damages, thus helping to extend the use of carbon-intensive capital in the green transition.

6 Conclusion

We have examined theoretically and empirically a mechanism that might explain why carbon-intensive capital is not run down in the transition to green growth. Without accumulating clean capital to offset carbon emissions and global warming damages, stringent climate regulations put a tighter constraint on the stock of carbon-intensive capital, so that this capital has to be run down over time.

In contrast, stepping-up clean capital accumulation, as induced by climate regulations, might not necessarily lead to running down of carbon-intensive dirty capital. The augmented stock of clean capital offsets global warming damages, protects the value of carbon-intensive capital, and avoids the running down of carbon-intensive capital. This provides more economic means to augment clean capital through investment and intersectoral capital reallocation. Through this mechanism, both carbon-intensive capital and carbon-saving clean capital can coexist.

The interplay between carbon-intensive and clean capital can generate an endogenous growth path along which both types of capital and consumption can enjoy sustained growth over the time horizon. The expanding stock of clean capital, by offsetting emissions caused by carbon-intensive dirty capital, flattens the growth trend of cumulative emissions and facilitates a transition to green endogenous growth.

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A Appendices

A.1 Implementing the Optimum in a Market Equilibrium

In the market equilibrium, the economy admits a representative household that maximizes $\int_0^\infty e^{-\rho t}U(C)dt$ subject to $\dot{K}_D = \pi + r_D K_D - C - I_C - R - \delta K_D$ and $\dot{K}_C = r_C K_C + I_C - \frac{\phi}{2}\frac{I_C^2}{K_C} + R - \frac{\kappa}{2}\frac{R^2}{K_C} - \delta K_C$. Here the household owns K_D and K_C, and receives remunerations by renting capital at the rate of return r_D and r_C, respectively. The household owns a representative firm using carbon-intensive and clean capital to produce final goods and receives profits π. The solution to the household problem is characterized as:

$$U'(C) = \lambda_D, \quad \lambda_D = (1-\phi I_C/K_C)\lambda_C, \quad \lambda_D = (1-\kappa R/K_C)\lambda_C, \tag{A.1a}$$

$$\rho\lambda_D - \dot{\lambda}_D = (r_D - \delta)\lambda_D, \quad \rho\lambda_C - \dot{\lambda}_C = \left(r_C + 0.5\phi(I_C/K_C)^2 + 0.5\kappa(R/K_C)^2 - \delta\right)\lambda_C. \tag{A.1b}$$

where characterizations of (A.1a) are the same as those of (11a)-(11c). A representative firm optimally chooses clean and carbon-intensive capital to maximize profit as: $\pi = F(K_D, K_C) - r_D K_D - \frac{r_C}{1-\phi\frac{I_C}{K_C}} K_C - \varepsilon P(K_D, K_C)$, where instantaneous profits π are obtained by subtracting the costs of renting carbon-intensive and clean capital and the cost of carbon emission abatement. The rental cost of carbon-intensive capital is r_D, and the rental cost of clean capital is r_C (the unit of clean capital is converted to final goods units by dividing $1-\phi\frac{I_C}{K_C}$). τ is the price charged for carbon emissions. Solving the firm's problem yields

$$r_D = F_{K_D} - \tau P_{K_D}, \qquad r_C = (F_{K_C} - \tau P_{K_C})\left(1 - \phi\frac{I_C}{K_C}\right). \tag{A.2}$$

Using the first expression of (A.2) to replace r_D and r_C in (A.1b), we obtain

$$\rho\lambda_D - \dot{\lambda}_D = (F_{K_D} - \delta)\lambda_D - \tau\lambda_D P_{K_D}, \quad \rho\lambda_C - \dot{\lambda}_C = F_{K_C}\lambda_D - \tau P_{K_C}\lambda_D + \left(\frac{\phi}{2}\left(\frac{I_C}{K_C}\right)^2 + \frac{\kappa}{2}\left(\frac{R}{K_C}\right)^2 - \delta\right)\lambda_C \tag{A.3}$$

Verify that by setting the emission pricing at a level of $\tau = \frac{-\lambda_S}{U'(C)}$, the equation (A.3) governing the shadow value of capital stocks in the market equilibrium is equivalent to characterizations of optimal allocations by (11d)-(11e). Here λ_S is the shadow cost of carbon and the corresponding HJB equation is (11f).

A.2 Proof of Proposition 1

The stable saddle path of the transitional dynamics of carbon-intensive capital is determined by

$$K_D(t) = K_D(0)e^{-|\mu|t} + K_D^*\left(1 - e^{-|\mu|t}\right) = K_D^0 e^{-|\mu|t} + k^*\bar{K}_C\left(1 - e^{-|\mu|t}\right), \tag{A.4}$$

where dirty capital $K_D(t)$ starts from the initial condition $K_D(0)=K_D^0$ and converges to the long-run steady state $K_D^*=k^*\bar{K}_C$. For the ease of analytical exposition, we introduce a term of carbon decay σS in the right-hand side of cumulative emission equation (17a) to allow an analytical expression of the long-run steady state. As the rate of decay σ is considered to be sufficiently small, introducing the term of carbon decay will not affect the fundamentals of cumulative emissions. In this case, the steady state of carbon-intensive capital, cumulative emissions, and their shadow values are determined by

$$K_D^*=k^*\bar{K}_C, \quad \lambda_D^*=\left((\eta(k^*)^{\alpha-1}-\delta)k^*\bar{K}_C\right)^{-1}, \quad S^*=\frac{\xi}{\sigma}(k^*)^{\beta}, \quad \lambda_S^*=-\frac{\epsilon\zeta^2\xi\chi}{(\rho+\epsilon)\sigma(\rho+\sigma)}(k^*)^{\beta}. \tag{A.5}$$

The dirty-clean capital ratio $k^*:=\frac{K_D^*}{\bar{K}_C}$ is determined by the stationary conditions of (17), i.e.,

$$\eta\alpha(k^*)^{\alpha-1}-\frac{\epsilon\zeta^2\xi^2\beta\chi}{(\rho+\epsilon)\sigma(\rho+\sigma)}(k^*)^{2\beta}\left(\eta(k^*)^{\alpha-1}-\delta\right)=\rho+\delta. \tag{A.6}$$

Given that $\eta\alpha(k^*)^{\alpha-1}>\rho+\delta$ and $\alpha<1$, we have $\eta(k^*)^{\alpha-1}>\frac{\rho+\delta}{\alpha}>\delta$. Then it is straightforward to verify that the left-hand side of (A.6) is monotonically decreasing in k^*. A tightening of regulations, higher χ, thus leads to a smaller ratio, k^*, and a smaller stock, K_D^*, with a given amount of clean capital $\bar{K}_C$.

Furthermore, the speed of transitional dynamics depends on the negative eigenvalue μ which is determined by

$$\mu=\frac{\rho}{2}-\sqrt{\left(\frac{\rho}{2}\right)^2-\frac{\Omega}{2}+\frac{1}{2}\sqrt{\Omega^2-4\det J}}. \tag{A.7}$$

where $\det J$ is the determinant of the 4×4 Jacobian matrix corresponding to (17) given by,

$$\det J\equiv\begin{vmatrix} \frac{\partial\dot{K}_D}{\partial K_D} & \frac{\partial\dot{K}_D}{\partial S} & \frac{\partial\dot{K}_D}{\partial\lambda_D} & \frac{\partial\dot{K}_D}{\partial\lambda_S} \\ \frac{\partial\dot{S}}{\partial K_D} & \frac{\partial\dot{S}}{\partial S} & \frac{\partial\dot{S}}{\partial\lambda_D} & \frac{\partial\dot{S}}{\partial\lambda_S} \\ \frac{\partial\dot{\lambda}_D}{\partial K_D} & \frac{\partial\dot{\lambda}_D}{\partial S} & \frac{\partial\dot{\lambda}_D}{\partial\lambda_D} & \frac{\partial\dot{\lambda}_D}{\partial\lambda_S} \\ \frac{\partial\dot{\lambda}_S}{\partial K_D} & \frac{\partial\dot{\lambda}_S}{\partial S} & \frac{\partial\dot{\lambda}_S}{\partial\lambda_D} & \frac{\partial\dot{\lambda}_S}{\partial\lambda_S} \end{vmatrix}=\begin{vmatrix} \frac{\partial\dot{K}_D}{\partial K_D} & 0 & \frac{\partial\dot{K}_D}{\partial\lambda_D} & 0 \\ \frac{\partial\dot{S}}{\partial K_D} & \frac{\partial\dot{S}}{\partial S} & 0 & 0 \\ \frac{\partial\dot{\lambda}_D}{\partial K_D} & 0 & \frac{\partial\dot{\lambda}_D}{\partial\lambda_D} & \frac{\partial\dot{\lambda}_D}{\partial\lambda_S} \\ 0 & \frac{\partial\dot{\lambda}_S}{\partial S} & 0 & \frac{\partial\dot{\lambda}_S}{\partial\lambda_S} \end{vmatrix} \tag{A.8}$$

and $\Omega=\Omega_1+\Omega_2+\Omega_3+\Omega_4$ with

$$\Omega_1=\begin{vmatrix} \frac{\partial\dot{K}_D}{\partial K_D} & \frac{\partial\dot{K}_D}{\partial\lambda_D} \\ \frac{\partial\dot{\lambda}_D}{\partial K_D} & \frac{\partial\dot{\lambda}_D}{\partial\lambda_D} \end{vmatrix}, \quad \Omega_2=\begin{vmatrix} \frac{\partial\dot{S}}{\partial S} & \frac{\partial\dot{S}}{\partial\lambda_S} \\ \frac{\partial\dot{\lambda}_S}{\partial S} & \frac{\partial\dot{\lambda}_S}{\partial\lambda_S} \end{vmatrix}, \quad \Omega_3=\begin{vmatrix} \frac{\partial\dot{K}_D}{\partial S} & \frac{\partial\dot{K}_D}{\partial\lambda_S} \\ \frac{\partial\dot{\lambda}_D}{\partial S} & \frac{\partial\dot{\lambda}_D}{\partial\lambda_S} \end{vmatrix}, \quad \Omega_4=\begin{vmatrix} \frac{\partial\dot{S}}{\partial K_D} & \frac{\partial\dot{S}}{\partial\lambda_D} \\ \frac{\partial\dot{\lambda}_S}{\partial K_D} & \frac{\partial\dot{\lambda}_S}{\partial\lambda_D} \end{vmatrix}.$$

Here Ω_1, Ω_2, Ω_3 and Ω_4 determine the dynamic stability of the subsystems. Evaluating $\det J$ given by

(A.8) at the steady state $[K_D^*, S^*, \lambda_D^*, \lambda_S^*]$ yields

$$\det J = \frac{\partial \dot{\lambda}_S}{\partial \lambda_S}\frac{\partial \dot{S}}{\partial S}\left(\frac{\partial \dot{K}_D}{\partial K_D}\frac{\partial \dot{\lambda}_D}{\partial \lambda_D} - \frac{\partial \dot{K}_D}{\partial \lambda_D}\frac{\partial \dot{\lambda}_D}{\partial K_D}\right) - \frac{\partial \dot{\lambda}_S}{\partial S}\frac{\partial \dot{S}}{\partial K_D}\frac{\partial \dot{K}_D}{\partial K_D}\frac{\partial \dot{\lambda}_D}{\partial \lambda_S}, \tag{A.9}$$

where $\frac{\partial \dot{\lambda}_S}{\partial \lambda_S} = \rho + \sigma$ and $\frac{\partial \dot{S}}{\partial S} = \sigma$. We can also show that $\frac{\partial \dot{K}_D}{\partial K_D}\frac{\partial \dot{\lambda}_D}{\partial \lambda_D} = -\left(\eta\alpha(k^*)^{\alpha-1} - \delta\right)\left(\eta\alpha(k^*)^{\alpha-1} - \delta - \rho\right)$, $\frac{\partial \dot{K}_D}{\partial \lambda_D}\frac{\partial \dot{\lambda}_D}{\partial K_D} = \alpha(1-\alpha)\eta(k^*)^{\alpha-1}\left(\eta(k^*)^{\alpha-1} - \delta\right) + \frac{\epsilon\zeta^2\xi\chi\beta(\beta-1)}{(\rho+\epsilon)\sigma(\rho+\sigma)}(k^*)^{2\beta}\left(\eta(k^*)^{\alpha-1} - \delta\right)^2$, and $\frac{\partial \dot{\lambda}_S}{\partial S}\frac{\partial \dot{S}}{\partial K_D}\frac{\partial \dot{K}_D}{\partial K_D}\frac{\partial \dot{\lambda}_D}{\partial \lambda_S} = \frac{\epsilon\zeta^2}{\rho+\epsilon}\xi\chi\beta^2(k^*)^{2\beta}\left(\eta(k^*)^{\alpha-1} - \delta\right)^2$. Substituting into (A.9) and simplifying the terms yields

$$\begin{aligned}\det J = & -\sigma(\sigma+\rho)\left[\left(\alpha\eta(k^*)^{\alpha-1} - \delta\right)\left(\alpha\eta(k^*)^{\alpha-1} - \delta - \rho\right) + \alpha(1-\alpha)\eta(k^*)^{\alpha-1}\left(\eta(k^*)^{\alpha-1} - \delta\right)\right] \\ & - \frac{\epsilon\zeta^2}{\rho+\epsilon}\xi\chi\beta(2\beta-1)(k^*)^{2\beta}\left(\eta(k^*)^{\alpha-1} - \delta\right)^2 < 0.\end{aligned} \tag{A.10}$$

Here due to $\alpha\eta(k^*)^{\alpha-1} > \rho + \delta$, $0 < \alpha < 1$ and $2\beta > 1$ (A.10) has a negative sign, implying a stable saddle path for the transitional dynamics. It is straightforward to verify that an increase in χ (stringent climate regulations) leads to a decrease in k^* and a lower $\det J$ and a higher $|\xi|$ as implied by (A.7). According to (A.4), a larger value of $|\xi|$ generates fast convergence of transitional dynamics. Accordingly, for a tightening of regulations, an increase in χ leads to a decrease in the long-run steady state K_D^* and an increase in the speed of convergence $|\xi|$. The transitional dynamic path determined by (A.4) implies that the stock of carbon-intensive capital is smaller at each time point when pollution regulations become stringent.

A.3 Proof of Proposition 2

To show that there is always a positive amount of capital investment and reallocation in clean sectors, we verify that it is impossible not to have clean capital investment and reallocation throughout the entire time frame. We prove it by contradiction. Suppose that there is no investment and reallocation over the entire time frame, i.e., $I_C(t) = 0$ and $R(t) = 0$, $\forall t \in [t_0, \infty)$ always holds. The efficiency conditions become $\lambda_C\left(1 - \phi\frac{I_C}{K_C}\right) < \lambda_D$ and $\lambda_C\left(1 - \kappa\frac{I_C}{K_C}\right) < \lambda_D$, which boil down to

$$\int_t^\infty e^{-(\rho+\delta)(s-t)}\left[U'(s)(F_{K_D}(s) - F_{K_C}(s)) + \lambda_S(s)(P_{K_D}(s) - P_{K_C}(s))\right]ds > 0, \tag{A.11}$$

To find the contradiction, we consider that at some time point, say t^*, that corresponds to the steady state, (A.11) is thus rewritten as $(\rho+\delta)^{-1}[U'(t^*)(F_{K_D}(t^*) - F_{K_C}(t^*)) + \lambda_S(t^*)(P_{K_D}(t^*) - P_{K_C}(t^*))] > 0$. Substituting the specific functions and simplifying yields

$$\frac{K_D^*}{C^*}\eta\left(\alpha(k^*)^{\alpha-1} - (1-\alpha)(k^*)^\alpha\right) > \frac{\epsilon\zeta^2\xi\chi\beta(k^*)^\beta}{(\rho+\epsilon)\sigma(\rho+\sigma)}\left((k^*)^\beta + (k^*)^{\beta+1}\right), \tag{A.12}$$

where C^* and K_D^* are consumption and carbon-intensive capital, respectively. $\bar{K}_C$ is the fixed stock of clean capital, $k^* := \frac{K_D^*}{\bar{K}_C}$, and $\lambda_S(t^*) = -\frac{\epsilon\zeta^2\xi\chi}{(\rho+\epsilon)\sigma(\rho+\sigma)}(k^*)^\beta$. It is easy to verify that the left-hand side of (A.12) is decreasing in k^* and becomes negative when $k^* > \frac{\alpha}{1-\alpha}$. But the right-hand side of (A.12) always has a positive sign, thus contracting with the inequality in (A.12).

Furthermore, we examine the effect of clean capital accumulation by considering a discrete increase in clean capital, say from $\bar{K}_C$ to $\bar{K}'_C$ with $\bar{K}'_C > \bar{K}_C$, and looking at how this change affects dirty capital. As (A.7)-(A.10) shows, the speed of transitional dynamics depends on the ratio $k^* := K_D^*/\bar{K}_C$ that is endogenously determined by a set of e parameters $[\alpha,\beta,\chi,\rho,\sigma,\delta,\epsilon,\zeta]$ as in (A.6), so changes in $\bar{K}_C$ have no effect on the speed of transitional dynamics. However, given a certain level of $k^* := K_D^*/\bar{K}_C$, dirty capital tends to reach a larger stock K_D^* when $\bar{K}_C$ increases. Therefore, over the entire transitional dynamics, an increase in clean capital generates an effect to increase carbon-intensive capital accumulation.

A.4 Proof of Proposition 3

Following the efficiency conditions given in (11), transitional dynamics are determined by the differential equations of $[K_D, K_C, S, T, \lambda_D, \lambda_C, \lambda_S, \lambda_T]$:

$$\frac{\dot{K}_D}{K_D} = \eta\left(\frac{K_D}{K_C}\right)^{\alpha-1} - \frac{1}{\lambda_D K_D} - \frac{\phi+\kappa}{\phi\kappa}\left(1-\frac{\lambda_D}{\lambda_C}\right)\left(\frac{K_C}{K_D}\right) - \delta, \qquad \frac{\dot{K}_C}{K_C} = \frac{\phi+\kappa}{\phi\kappa}\left(1-\frac{\lambda_D}{\lambda_C}\right) - \frac{\phi+\kappa}{2\phi\kappa}\left(1-\frac{\lambda_D}{\lambda_C}\right)^2 - \delta,$$
$$\dot{S} = \xi\left(\frac{K_D}{K_C}\right)^\beta, \qquad \frac{\dot{\lambda}_D}{\lambda_D} = \rho+\delta-\eta\alpha\left(\frac{K_D}{K_C}\right)^{\alpha-1} - \xi\beta\lambda_S\left(\frac{K_D}{K_C}\right)^\beta \frac{1}{\lambda_D K_D},$$
$$\frac{\dot{\lambda}_C}{\lambda_C} = \rho+\delta-(1-\alpha)\left(\frac{K_D}{K_C}\right)^\alpha \frac{\lambda_D}{\lambda_C} + \xi\beta\lambda_S\left(\frac{K_D}{K_C}\right)^\beta \frac{1}{\lambda_C K_C} - \frac{\phi+\kappa}{2\phi\kappa}\left(1-\frac{\lambda_D}{\lambda_C}\right)^2, \qquad \dot{\lambda}_S = \rho\lambda_S + \chi S,$$
$$\dot{T} = \epsilon(\zeta S - T), \qquad \dot{\lambda}_T = (\rho+\epsilon)\lambda_T + \chi T.$$

We derive characterizations of endogenous growth as follows.

First, following the aggregate resource constraint and the law of motion for capital stocks, we have

$$\frac{I_C}{K_C} = \frac{\eta K_D^\alpha K_C^{1-\alpha}}{K_C} - \frac{C}{K_D}\frac{K_D}{K_C} - \frac{\dot{K}_D + R + \delta K_D}{K_C} = \eta k^\alpha - (c+g+\delta)k - \frac{R}{K_C}. \tag{A.13}$$

Following the efficiency condition, we have $\frac{R}{K_C} = \left(\frac{\phi}{\kappa}\right)\frac{I_C}{K_C}$ and

$$\frac{I_C}{K_C} = \frac{\kappa}{\kappa+\phi}(\eta k^\alpha - (c+g+\delta)k), \quad \frac{R}{K_C} = \frac{\phi}{\kappa+\phi}(\eta k^\alpha - (c+g+\delta)k). \tag{A.14}$$

Then, rewriting (8b) yields $\frac{\dot{K}_C}{K_C} = \frac{I_C}{K_C} + \frac{R}{K_C} - \frac{\phi}{2}\left(\frac{I_C}{K_C}\right)^2 - \frac{\kappa}{2}\left(\frac{R}{K_C}\right)^2 - \delta$, and using (A.14) to replace $\frac{I_C}{K_C}$ and

$\frac{R}{K_C}$, we obtain (20). Second, following (11a) and (11d), we derive the Euler equation of consumption

$$\frac{\dot{C}}{C} = F_{K_D} - \rho - \delta + \frac{\lambda_S P_{K_D}}{U'} = \eta\alpha k^{\alpha-1} - \rho - \delta + \xi\beta\lambda_S c k^\beta, \tag{A.15}$$

where the marginal pollution damages of carbon-intensive capital are given by $\frac{\lambda_S P_{K_D}}{U'} = C\lambda_S \xi\beta K_D^{\beta-1} K_C^{-\beta} = \xi\beta\lambda_S \frac{C}{K_D}\left(\frac{K_D}{K_C}\right)^\beta = \xi\beta\lambda_S c k^\beta$. Rewriting yields (21). Third, following (11b), (11d) and (11e), we obtain the efficiency condition for carbon-intensive and clean capital investment:

$$F_{K_D} + \frac{\lambda_S P_{K_D}}{U'} = \left(F_{K_C} + \frac{\lambda_S P_{K_C}}{U'}\right)\left(1 - \phi\frac{I_C}{K_C}\right) + \frac{\phi}{2}\left(\frac{I_C}{K_C}\right)^2 + \frac{\kappa}{2}\left(\frac{R}{K_C}\right)^2, \tag{A.16}$$

where the left- and right-hand side corresponds to marginal benefits of accumulating carbon-intensive and clean capital, respectively. Simplifying yields (22).

A.5 Numerical Implementation in Section 4

Numerical simulations of green endogenous growth are implemented in the follow steps:

Step 1. Imposing stationery conditions on (20)-(26), the long-run balanced growth path (BGP) is determined by

$$g^* = \eta(k^*)^{\alpha-1} - c^* - \frac{\phi+\kappa}{\phi\kappa}\left(1 - \sqrt{1 - \frac{2\phi\kappa(\delta+g^*)}{\phi+\kappa}}\right)(k^*)^{-1} - \delta, \tag{A.17a}$$

$$g^* = \eta\alpha(k^*)^{\alpha-1} - \frac{\beta\epsilon\zeta^2\xi^2\chi}{(\rho+\sigma)\sigma(\rho+\epsilon)}c^*(k^*)^{2\beta} - \rho - \delta, \tag{A.17b}$$

$$g^* = \frac{\phi+\kappa}{2\phi\kappa}\left(1 - \sqrt{1 - \frac{2\phi\kappa(\delta+g^*)}{\phi+\kappa}}\right)^2 + \left(\eta(1-\alpha)(k^*)^\alpha + \frac{\beta\epsilon\zeta^2\xi^2\chi}{(\rho+\sigma)\sigma(\rho+\epsilon)}c^*(k^*)^{2\beta+1}\right)\sqrt{1 - \frac{2\phi\kappa(\delta+g^*)}{\phi+\kappa}} - \rho - \delta, \tag{A.17c}$$

In deriving (A.17), we use the BGP levels of cumulative emissions, temperature increases, the shadow costs of carbon and warming, and the ratio of shadow prices between carbon-intensive and clean capital as follows:

$$S^* = \frac{\xi}{\sigma}(k^*)^\beta, \qquad \lambda_S^* = \frac{\epsilon\zeta}{(\rho+\sigma)}\lambda_T^* = -\frac{\epsilon\zeta^2\chi\xi}{(\rho+\sigma)\sigma(\rho+\epsilon)}(k^*)^\beta, \qquad T^* = \zeta S^* = \frac{\zeta\xi}{\sigma}(k^*)^\beta, \tag{A.18a}$$

$$\lambda_T^* = -\frac{\chi}{\rho+\epsilon}T^* = -\frac{\chi\zeta\xi}{\sigma(\rho+\epsilon)}(k^*)^\beta, \qquad \frac{\lambda_D^*}{\lambda_C^*} = \sqrt{1 - \frac{2\phi\kappa(\delta+g^*)}{\phi+\kappa}}. \tag{A.18b}$$

The shadow prices of carbon-intensive and clean capital change at the same rate in the long run, $\dot{\lambda}_C^*/\lambda_C^* = \dot{\lambda}_D^*/\lambda_D^* = -\dot{C}^*/C^* = -g^*$. Solving the three equations in (A.17) for three endogenous variables

$[k^*, c^*, g^*]$. Then calculate $[S^*, T^*, \lambda_S^*, \lambda_T^*]$ using (A.18).

Step 2. Following (20)-(24), we simulate the transitional dynamic path by solving the following system of differential equations:

$$\frac{\dot{k}}{k} = \frac{\phi\kappa}{2(\phi+\kappa)}(\eta k^\alpha - (c + \mathrm{g}(c,k,S,\lambda_S)+\delta)k)^2 - (\eta k^\alpha - (c+\mathrm{g}(c,k,S,\lambda_S)+\delta)k) + \delta + \mathrm{g}(c,k,S,\lambda_S), \quad \text{(A.19a)}$$

$$\frac{\dot{c}}{c} = \eta\alpha k^{\alpha-1} + \xi\beta\lambda_S c k^\beta - \rho - \delta - \mathrm{g}(c,k,S,\lambda_S), \quad \text{(A.19b)}$$

$$\dot{S} = \xi k^\beta - \sigma S, \quad \text{(A.19c)}$$

$$\dot{\lambda}_S = (\rho+\sigma)\lambda_S - \epsilon\zeta\lambda_T, \quad \text{(A.19d)}$$

$$\dot{T} = \epsilon(\zeta S - T), \quad \text{(A.19e)}$$

$$\dot{\lambda}_T = (\rho+\epsilon)\lambda_T + \chi T; \quad \text{(A.19f)}$$

where g is determined by the non-arbitrage condition (22) as a function of $[c,k,S,\lambda_S]$:

$$g = \mathrm{g}(c,k,S,\lambda_S) = \eta\alpha k^{\alpha-1} + \xi\beta\lambda_S c k^\beta - c - \delta + \sqrt{(\eta(1-\alpha)k^{\alpha-1} - \xi\beta\lambda_S c k^\beta)^2 - \frac{2(\phi+\kappa)}{\phi\kappa}[\eta k^{\alpha-3}((1-\alpha)k-\alpha) - \xi\beta\lambda_S c k^{\beta-2}(k+1)]}. \quad \text{(A.20)}$$

Step 3. Given $[k,c]$ numerically solved above, compute the growth rate of carbon-intensive capital using $\dot{K}_D/K_D \coloneqq g = \mathrm{g}(k,c)$ given in (A.20), then compute the growth rate of clean capital, consumption and production using $\dot{K}_C/K_C = g - \dot{k}/k$, $\dot{C}/C = g + \dot{c}/c$ and $\dot{Y}/Y = \alpha g + (1-\alpha)(g - \dot{k}/k)$.

Step 4. Given the initial condition $K_D(0)$ and the growth rate of carbon-intensive capital $g(t)$, compute the time path of carbon-intensive capital stocks using $K_D(t) = K_D(0)\exp\left(\int_0^t g(t)dt\right)$,

Step 5. Given $K_D(t)$, $k(t)$ and $c(t)$, compute the time paths of clean capital and consumption using $K_C(t) = k(t)K_D(t)$ and $C(t) = c(t)K_D(t)$.

Step 6. Compute the amount of clean capital investment and intersectoral capital reallocation as follows: $I_C(t) = \frac{\kappa}{\kappa+\phi}(\eta k(t)^\alpha - (c(t)+g(t)+\delta)k(t))K_C(t)$ and $R(t) = \frac{\phi}{\kappa+\phi}(\eta k(t)^\alpha - (c(t)+g(t)+\delta)k(t))K_C(t)$.

A.6 Flow-related Climate Damages

The Running Down of Carbon-Intensive Capital. In the case where rate-related climate damages are attributable to the flow of carbon emissions, we consider the following problem that maximizes

$$\max_{[C(t), I_D(t), R(t)]_{t=0}^\infty} \int_0^\infty e^{-\rho t}\left(\ln C(t) - \frac{1}{2}\chi E(t)^2\right)dt, \quad \text{(A.21)}$$

subject to the law of motion for carbon-intensive capital (8a) and clean capital (8b), given the initial conditions $K_D(0)=K_D^0$ and $K_C(0)=K_C^0$. The efficiency conditions are given by

$$C=\lambda_D^{-1}, \qquad I_C=\phi^{-1}(1-\lambda_D/\lambda_C)K_C, \qquad R=\kappa^{-1}(1-\lambda_D/\lambda_C)K_C, \tag{A.22a}$$

$$(\rho+\delta)\lambda_D-\dot{\lambda}_D=-V'P_{K_D}+U'F_{K_D}, \tag{A.22b}$$

$$(\rho+\delta)\lambda_C-\dot{\lambda}_C=-V'P_{K_C}+U'\left(F_{K_C}+\frac{0.5\phi(I_C/K_C)^2+0.5\kappa(R/K_C)^2}{1-\phi I_C/K_C}\right). \tag{A.22c}$$

Proposition A1. *When there is no accumulation of clean capital to offset flow-related climate damages, a tightening of climate regulations lead to the running down of carbon-intensive capital.*

Proof. Following the logic of Section 3, we consider the problem maximizing (9) subject to $\dot{K}_D(t)=\eta K_D(t)^\alpha \bar{K}_C^{1-\alpha}-C(t)-\delta K_D(t)$, given $K_D(0)=K_D^0$. Economic growth is only driven by carbon-intensive capital and clean capital remains fixed $K_C(t)=\bar{K}_C$. Solving the problem gives the characterizations of the transitional dynamics:

$$\dot{K}_D=\eta K_D^\alpha \bar{K}_C^{1-\alpha}-\lambda_D^{-1}-\delta K_D, \quad \dot{\lambda}_D=\left(\rho+\delta-\eta\alpha K_D^{\alpha-1}\bar{K}_C^{1-\alpha}\right)\lambda_D+\xi\beta\chi K_D^{2\beta-1}\bar{K}_C^{-2\beta}. \tag{A.23}$$

Imposing the long-run steady-state conditions on (A.23) yields

$$\eta\alpha(k^*)^{\alpha-1}-\xi\beta\chi(k^*)^{2\beta}\left(\eta(k^*)^{\alpha-1}-\delta\right)=\rho+\delta. \tag{A.24}$$

which endogenously determined $k^*:=K_D^*/\bar{K}_C$. Given k^*, we have $K_D^*=k^*\bar{K}_C$ and $\lambda_D^*=\left(\frac{\xi\chi\beta(k^*)^{2\beta-1}}{\eta\alpha(k^*)^{\alpha-1}-\rho-\delta}\right)\frac{1}{\bar{K}_C}$. Given that $\eta\alpha(k^*)^{\alpha-1}>\rho+\delta$ and $\alpha<1$, we have $\eta(k^*)^{\alpha-1}>\frac{\rho+\delta}{\alpha}>\delta$, so verify that the left-hand side of (A.24) is monotonically decreasing in k^*. Hence, a tightening of regulations, higher χ leads to drop in k^* and K_D^*.

Along the stable saddle path, the stock of carbon-intensive capital evolves according to

$$K_D(t)=K_D(0)e^{-|\mu|t}+K_D^*\left(1-e^{-|\mu|t}\right)=K_D^0e^{-|\mu|t}+k^*\bar{K}_C\left(1-e^{-|\mu|t}\right), \tag{A.25}$$

where $K_D(t)$ starts from the initial condition $K_D(0)=K_D^0$ and converges to the steady state $K_D^*=k^*\bar{K}_C$. The speed of convergence is determined by the negative eigenvalue $\mu=\left(\rho-\sqrt{\rho^2-4\det(J(K_D^*,\lambda_D^*))}\right)/2$, where the determinant of the Jacobian matrix is given by

$$\det(J(K_D^*,\lambda_D^*)) \equiv \begin{bmatrix} \frac{\partial \dot{K}_D}{\partial K_D} & \frac{\partial \dot{K}_D}{\partial \lambda_D} \\ \frac{\partial \dot{\lambda}_D}{\partial K_D} & \frac{\partial \dot{\lambda}_D}{\partial \lambda_D} \end{bmatrix}$$

$$= \left(\eta\alpha\left(\frac{K_D^*}{\bar{K}_C}\right)^{\alpha-1} - \delta\right)\left(\rho+\delta-\eta\alpha\left(\frac{K_D^*}{\bar{K}_C}\right)^{\alpha-1}\right) - \left[\left(\frac{K_D^*}{\bar{K}_C}\right)^{\alpha-1}\frac{\eta\alpha(1-\alpha)}{\lambda_D^* K_D^*} + \left(\frac{K_D^*}{\bar{K}_C}\right)^{2\beta}\frac{\xi\chi\beta(2\beta-1)}{(\lambda_D^* K_D^*)^2}\right] < 0,$$

Due to $\alpha\eta(K_D^*/\bar{K}_C)^{\alpha-1} > \rho+\delta$, $0<\alpha<1$, and $2\beta>1$, the negative sign implies a stable saddle path for transitional dynamics. A tightening of pollution regulations, higher χ, leads to lower k^* and $\det(J(K_D^*,\lambda_D^*))$, and higher $|\xi|$ and a faster speed of transitional dynamics. □

Proposition A2. *In the environment with flow-related climate damages, an increase in the stock of clean capital offsetting pollution damages lessens the degree of running down carbon-intensive capital caused by climate regulations.*

Proof. First, we show that it is impossible not to launch clean capital accumulation. Suppose that there is no capital investment and reallocation over the entire time frame, i.e., $I_C(t)=0$ and $R(t)=0$, $\forall t \in [t_0,\infty)$ always holds. The condition under which there is neither investment nor reallocation, i.e., $\lambda_C\left(1-\phi\frac{I_C}{K_C}\right) < \lambda_D$ and $\lambda_C\left(1-\kappa\frac{I_C}{K_C}\right) < \lambda_D$, boils down to

$$\int_t^{\infty} e^{-(\rho+\delta)(t'-t)}\left[U'(t')\left(F_{K_D}(t')-F_{K_C}(t')\right) - V'(t')\left(P_{K_D}(t')-P_{K_C}(t')\right)\right]dt' > 0, \tag{A.26}$$

To find the contradiction, we consider that at the steady state t^*, (A.26) is rewritten as

$$\frac{1}{C^*}\eta\left(\alpha\left(\frac{K_D^*}{\bar{K}_C}\right)^{\alpha-1} - (1-\alpha)\left(\frac{K_D^*}{\bar{K}_C}\right)^{\alpha}\right) > \xi\beta\chi\left(\frac{K_D^*}{\bar{K}_C}\right)^{\beta}\left(\left(\frac{K_D^*}{\bar{K}_C}\right)^{\beta}\frac{1}{K_D^*} + \left(\frac{K_D^*}{\bar{K}_C}\right)^{\beta}\frac{1}{\bar{K}_C}\right), \tag{A.27}$$

Verify that the left-hand side of (A.27) is decreasing in K_D and becomes negative when $\frac{K_D^*}{\bar{K}_C} > \frac{\alpha}{1-\alpha}$. But the right-hand side of (A.27) always has a positive sign, thus contracting with the inequality in (A.27). Second, consider a discrete increase in the stock of clean capital, say from $\bar{K}_C$ to $\bar{K}_C'$ with $\bar{K}_C' > \bar{K}_C$, and look at how this change affects carbon-intensive capital. As (A.25) show, the speed of convergence alone the transitional dynamic path depends on the dirty-clean capital ratio $k^* := K_D^*/\bar{K}_C$ that is determined by a set of parameters $[\eta,\xi,\alpha,\beta,\chi,\rho,\delta]$ as in (A.24), so changes in $\bar{K}_C$ have no effect on the speed of convergence. However, given a certain level of $k^* := K_D^*/\bar{K}_C$, dirty capital will converge towards a larger steady-state stock K_D^* when the stock of clean capital increases $\bar{K}_C$. Therefore, over the entire transitional dynamics, carbon-intensive capital has stronger growth when there is an increase in the stock of clean capital. □

Green Endogenous Growth. We proceed by characterizing green endogenous growth under flow-related climate damages, characterizations of the green endogenous growth are given by

Proposition A3. *The endogenous growth path as characterized by the triple* $[g,k,c]$ *is determined by the following system of equations:*

i. the law of motion for the ratio of carbon-intensive to clean capital, k:

$$\frac{\dot{k}}{k}=\frac{\phi\kappa}{2(\phi+\kappa)}(\eta k^{\alpha}-(c+g+\delta)k)^2-(\eta k^{\alpha}-(c+g+\delta)k)+\delta+g, \tag{A.28}$$

ii. the law of motion for the ratio of consumption to carbon-intensive capital, c:

$$\frac{\dot{c}}{c}=\eta\alpha k^{\alpha-1}-\xi\chi\beta ck^{2\beta}-\rho-\delta-g, \tag{A.29}$$

iii. the efficiency condition for carbon-intensive and clean capital investments:

$$\eta\alpha k^{\alpha-1}-\xi\chi\beta ck^{2\beta}=\left(\eta(1-\alpha)k^{\alpha}+\xi\chi\beta ck^{2\beta+1}\right)\left(1-\frac{\phi\kappa}{\kappa+\phi}(\eta k^{\alpha}-(c+g+\delta)k)\right)+\frac{\phi\kappa}{2(\kappa+\phi)}(\eta k^{\alpha}-(c+g+\delta)k)^2, \tag{A.30}$$

where $[g,k,c]$ *is defined by* $g:=\frac{\dot{K}_D}{K_D}$*,* $k:=\frac{K_D}{K_C}$*, and* $c:=\frac{C}{K_D}$*.*

Proof. This proposition can be proved analogously as in the proof of Proposition 3. □

We simulate growth under rate-related climate damages by numerically solving (A.28), (A.29), and (A.30). The procedures of numerical implementations include two steps. First, we impose stationery conditions on (A.28)-(A.30) and derive the following three equations that determine $[k^*,c^*,g^*]$

$$\begin{aligned}
&\eta(k^*)^{\alpha}-(c^*+g^*+\delta)k^*=\frac{\phi+\kappa}{\phi\kappa}\left(1-\sqrt{1-\frac{2(\delta+g^*)\phi\kappa}{\phi+\kappa}}\right),\\
&g^*+\rho+\delta=\eta\alpha(k^*)^{\alpha-1}-\xi\chi\beta c^*(k^*)^{2\beta},\\
&g^*+\rho+\delta=\left(\eta(1-\alpha)(k^*)^{\alpha}+\xi\chi\beta c^*(k^*)^{2\beta+1}\right)\sqrt{1-\frac{2(\delta+g^*)\phi\kappa}{\phi+\kappa}}+\frac{\kappa+\phi}{2\kappa\phi}\left(1-\sqrt{1-\frac{2(\delta+g^*)\phi\kappa}{\phi+\kappa}}\right)^2.
\end{aligned}$$

Second, simplifying (A.28)-(A.30), we describe the transitional dynamics by the system of differential equations of $[k,c]$:

$$\frac{\dot{k}}{k}=\frac{\phi\kappa}{2(\phi+\kappa)}(\eta k^{\alpha}-(c+\mathrm{g}(c,k)+\delta)k)^2-(\eta k^{\alpha}-(c+\mathrm{g}(c,k)+\delta)k)+\delta+\mathrm{g}(c,k), \tag{A.31a}$$

$$\frac{\dot{c}}{c}=\eta\alpha k^{\alpha-1}-\xi\chi\beta ck^{2\beta}-\rho-\delta-\mathrm{g}(c,k), \tag{A.31b}$$

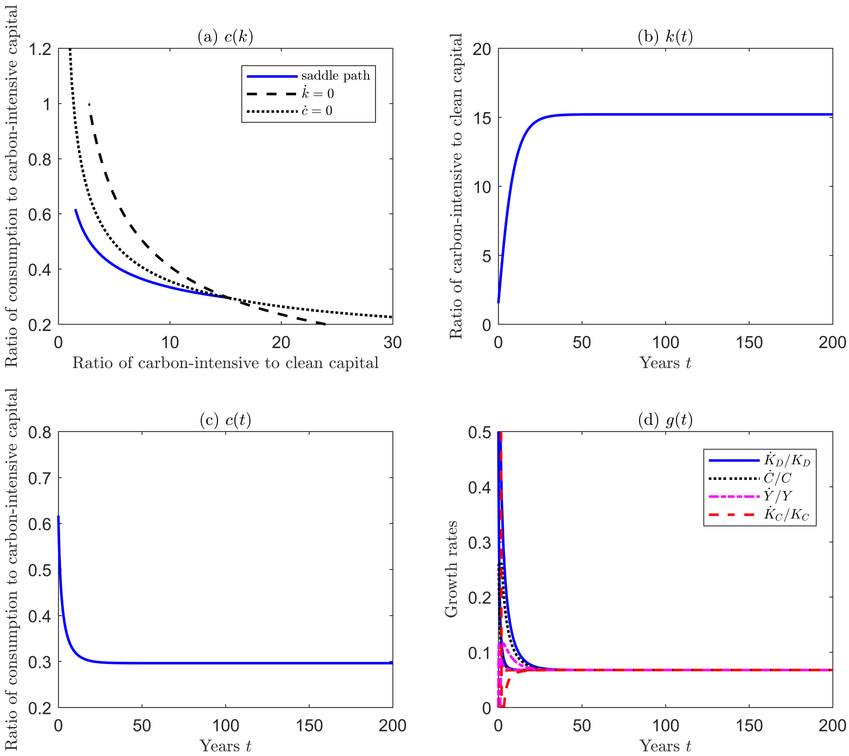

Figure 9: The Time Path of $[k, c, g]$ in Green Endogenous Growth under Flow-related Climate Damages. Panel (a) plots the phase diagram of green endogenous growth determined by (A.28)-(A.30). The solid blue line depicts the stable saddle path starting from the initial condition and converging to the long-run BGP. Panel (b) plots the time path of the ratio of carbon-intensive to clean capital k. Panel (c) plots the time path of the ratio of consumption to carbon-intensive capital. Panel (d) depicts the growth rate of carbon-intensive capital, clean capital, consumption and outputs.

where g is derived by (A.30) as a function of c and k:

$$g = \mathrm{g}(c,k) = \eta\alpha k^{\alpha-1} - \xi\chi\beta c k^{2\beta} - c - \delta$$
$$+ \sqrt{(\eta(1-\alpha)k^{\alpha-1} + \xi\chi\beta c k^{2\beta})^2 - \frac{2(\phi+\kappa)}{\phi\kappa}[\eta k^{\alpha-3}((1-\alpha)k-\alpha) + \xi\chi\beta c k^{2\beta-2}(1+k)]}.$$

Then using the BGP results obtained in the first step as the terminal condition, we numerically solve (A.31) and simulate the transitional dynamic path. Figure 9 shows the time paths of $[k,c,g]$ that characterize green endogenous growth determined by (A.28)-(A.30). Panel (a) shows the phase diagram of transitional dynamics, where $[k,c]$ evolves along the stable saddle path and converges to the long-run BGP. Panel (b) plots the time path of k: starting with the initial condition and converging to the BGP. Panel (c) depicts the time path of c that evolves from the initial condition and converges to the BGP. Panel (d) shows that

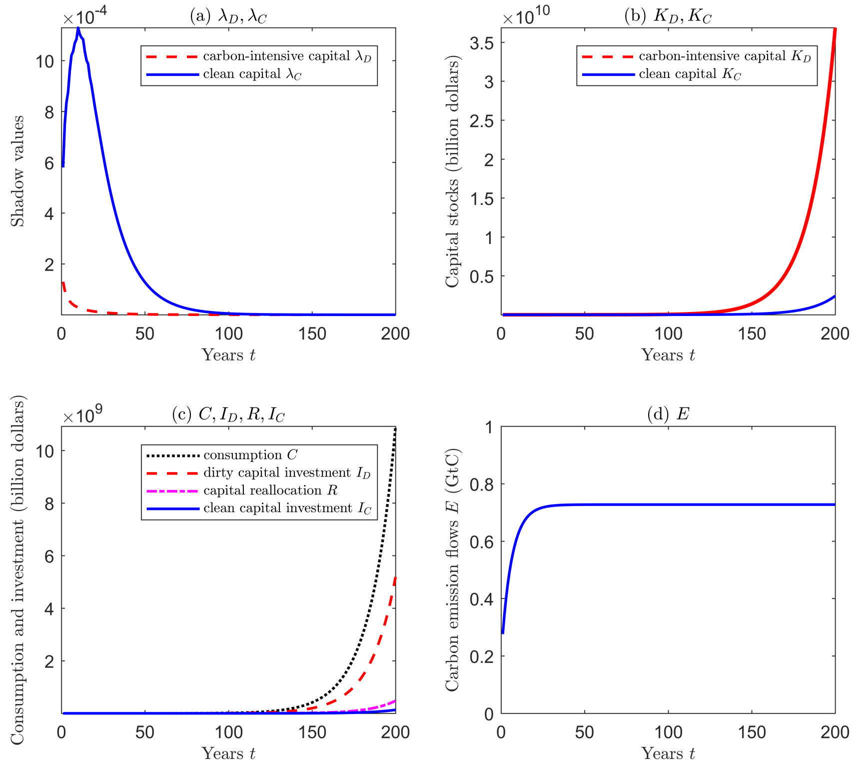

Figure 10: The Time Path of $[\lambda_D, \lambda_C, K_D, K_C, C, I_C, I_D, R, E]$ in Green Endogenous Growth under Flow-related Climate Damages. Panel (a) plots the transitional dynamic paths of shadow values $[\lambda_D, \lambda_C]$ where clean capital generates a larger shadow value. Panel (b) plots the time paths of carbon-intensive and clean capital $[K_D, K_C]$. Panel (c) plots the growth paths of consumption, capital investment, and intersectoral capital reallocation $[C, I_C, I_D, R]$. Panel (d) shows the time path of flow emissions E.

the growth rates of carbon-intensive capital, clean capital, consumption and outputs with convergence in the long run. In Figure 10, panel (a) shows that the endogenous growth path determines a larger shadow value of clean capital, which incentivizes both investment and reallocating capital goods from carbon-intensive to clean sectors. Panel (b) simulates the accumulation of both types of capital along the endogenous growth path. Panel (c) depicts the endogenous growth paths of consumption, capital investment and intersectoral capital reallocation. Panel (d) shows the time path of pollution emissions. As the expanding stock of clean capital, by offsetting the polluting effect of carbon-intensive capital, flattens the growth curve of emissions and creates convergence towards the steady-state levels in the long run.