International Conference on Analysis and Applications 2024

3 June – 6 June 2024, City University of Hong Kong

Program and Abstracts

Contents

Welcome Message

Welcome to City University of Hong Kong (CityU), the host of the International Conference on Analysis and Applications 2024 (ICAA 2024).

We are delighted to invite you to join us at this prestigious event, held in honor of Roderick S. C. Wong's 80th birthday. The conference provides a platform for researchers, scholars, and practitioners from around the world to come together and explore the latest advancements in the field of analysis and its applications.

Our objective is to foster collaboration, knowledge exchange, and intellectual dialogue among participants. Through engaging keynote speeches, insightful presentations, and interactive discussions, we aim to create a vibrant atmosphere that encourages the sharing of ideas and the formation of new partnerships.

In addition to the enriching academic program, we have also arranged social events and networking opportunities, allowing you to connect with fellow participants and experts in a relaxed and inspiring setting. You will also have the chance to explore the vibrant city of Hong Kong, known for its captivating blend of tradition and modernity.

We extend our warmest welcome to all attendees, including researchers, students, industry professionals, and anyone passionate about the field of analysis. Your presence and contributions will undoubtedly enhance the success of this conference and contribute to the advancement of knowledge in the field.

General Information

Organizing Committee

Dan DAI , City University of Hong Kong, Hong Kong (Chair)
Ya Yan LU, City University of Hong Kong, Hong Kong
Chunhua OU, Memorial University of Newfoundland, Canada
Lun ZHANG, Fudan University, China
Dingxuan ZHOU, The University of Sydney, Australia

Conference Coordinators

Conference Secretary
Nina CHAN
Liu Bie Ju Centre for Mathematical Sciences, City University of Hong Kong

Website: https://www.cityu.edu.hk/rcms/icaa2024/index.html

REGISTRATION

Date: 3 June to 6 June (Monday to Thursday)
Time: 9:00 – 9:30 am (Monday, 3 June)
8:30 am - 9:00 am (Tuesday, 4 June to 6 June)

CONFERENCE VENUE

Leung Ko Yuk Tak Lecture Theatre (LT-14),4/F, Yeung Kin Man Academic Building, CityU

LT-14 is equipped with a desktop computer, a cable for connecting to laptop, an overhead projector and white boards.

SOCIAL EVENTS

Welcome Reception (by Invitation Only)
Date: 3 June 2024 (Mon)
Time: 6:30 pm
Venue: CityU Lodge, Academic Exchange Building, 81 Tat Chee Avenue

Conference Banquet (by Invitation Only)
Date: 5 June 2024 (Wed)
Time: 6:30 pm
Venue: City Chinese Restaurant
8/F, Bank of China (Hong Kong) Complex, City University of Hong Kong

HK$600 for extra banquet ticket

Other Information

Name Badges

All attendees are requested to wear their name badge. Conference Secretary and Conference Assistants are ready to assist you if needed.

Banking Service

Opening hours: 09:00–17:00 (Monday–Friday)
Location: 3/F Yeung Kin Man Academic Building (next to Run Run Shaw Library)

Services including foreign currency and traveler's cheque exchange are provided at Hang Seng bank.

Message Board

Message boards are located outside Lecture Theatre 14 (LT-14). The latest information of the conference and messages for attendees will be posted on these boards.

Computer & Internet Services

Networked computers are available at Mathematical Laboratory during the conference period:
Date: 3 June to 6 June 2024
Time: 09:00-12:30, 13:45-17:30
Venue: Y6504, 6/F, Yeung Kin Man Academic Building (Yellow Zone near Lift 9)
Please contact our colleague to unlock the door of the computer lab for you.

Wireless internet access through your own mobile device within CityU campus is also available.
Login name and password can be found at the inner side of your name badge.

Dining

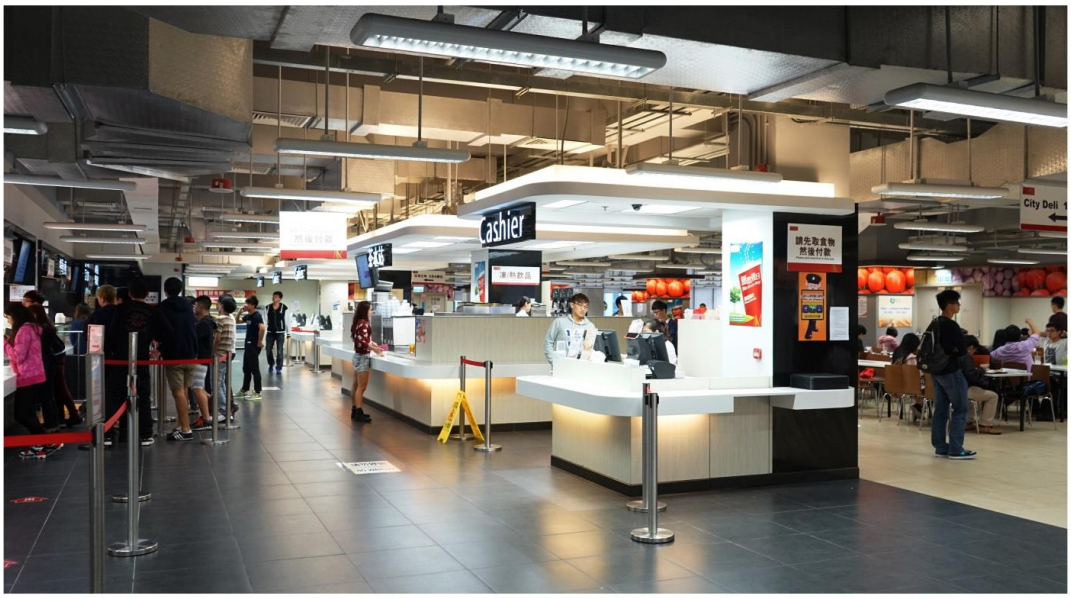
Several canteens are available at the campus and over 30 restaurants can be found at the adjacent shopping mall Festival Walk.

CityU Campus

City Express

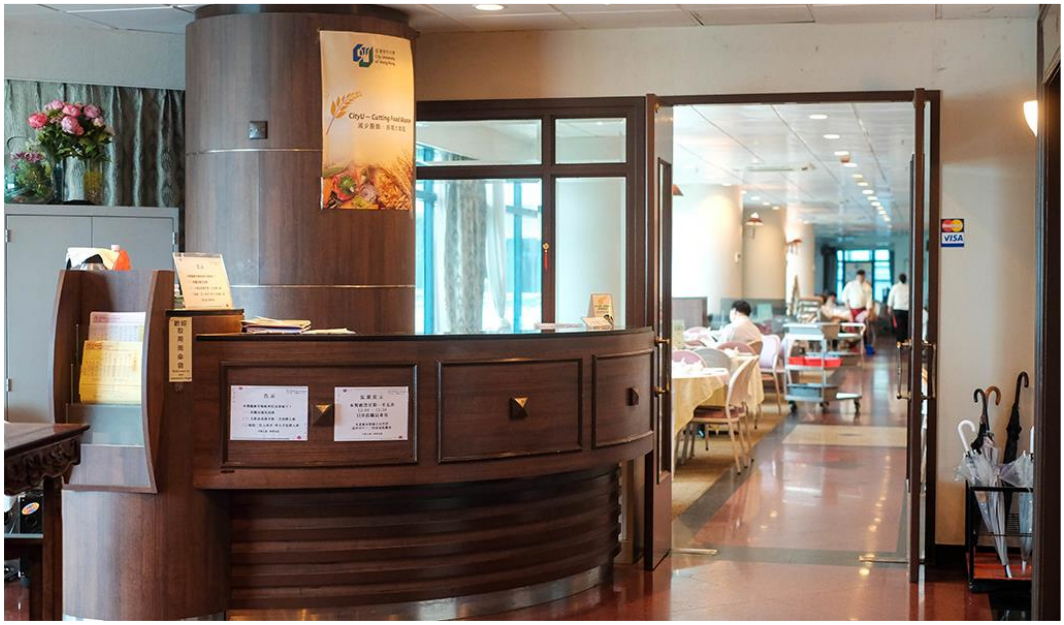
Location: 5/F, Yeung Kin Man Academic Building
Opening hours: 07:30–21:00 (Monday–Sunday)
Menu: Fast food, dim sum, short orders, set meal, kebab, vegetarian dish, daily carving, drinks and bakery products.

AC2 Canteen



Location: 3/F, Li Dak Sum Yip Yio Chin Academic Building
Opening hours: 07:30–21:00 (Monday–Sunday)
Menu: Fast food

City Chinese Restaurant



Location: 8/F, Bank of China (Hong Kong) Complex
Opening hours: 11:00–15:50 & 17:30-21:30 (Monday–Friday)
09:00–15:50 & 17:30-21:30 (Saturday, Sunday and public holidays)
Menu: Chinese menu with full selection

Faculty Lounge

Location: 9/F, Bank of China (Hong Kong) Complex
Opening hours: 11:00–22:30 (Monday–Sunday)
Menu: Western menu

AC3 Bistro

Location: 7/F, Lau Ming Wai Academic Building
Opening hours: 07:30–21:00 (Monday–Saturday)
Closed on Sunday and public holidays
Menu: Western food

AC3 Café

Location: 3/F, Lau Ming Wai Academic Building
Opening hours: 07:30–21:00 (Monday–Friday)
09:00–19:00 (Saturday and Sunday)
Closed on public holidays
Menu: Sandwich, salad, snacks and drinks

Lodge Bistro

Location: G/F, Academic Exchange Building
Opening hours: 07:30–20:30 (Monday–Sunday)
Menu: Western food

5380 Cafe (Kebab Station)

Location: 5/F, Bank of China (Hong Kong) Complex
Opening hours: 10:00–20:00 (Monday–Saturday)
Closed on Sunday and public holidays
Menu: Hot halal food and kebab

Coffee Cart
Location: Purple Zone, 4/F, Yeung Kin Man Academic Building
Opening hours: 08:00–20:00 (Monday–Friday)
08:00–17:00 (Saturday)
Closed on Sundays and public holidays
Service: Snacks, drinks and stationeries

Homey Kitchen
Location: Student Residence Multi-function Hall B
Opening hours: 9:30 – 15:15 & 15:45 – 19:30 (Monday – Friday)
11:00 – 16:00 (Saturday)
Closed on Sundays and public holidays
Service: Snacks, drinks and stationeries

Festival Walk (Shopping Mall)
With an extensive selection of restaurants and menus, this adjacent shopping mall provides more choices for dinning. The dinning and shopping directories of the mall can be found in the conference package.

Useful Telephone Numbers

CityU Campus

LBJ Centre:	+852 3442 6570
Health Centre:	+852 3442 6066
Security Office:	+852 3442 8888

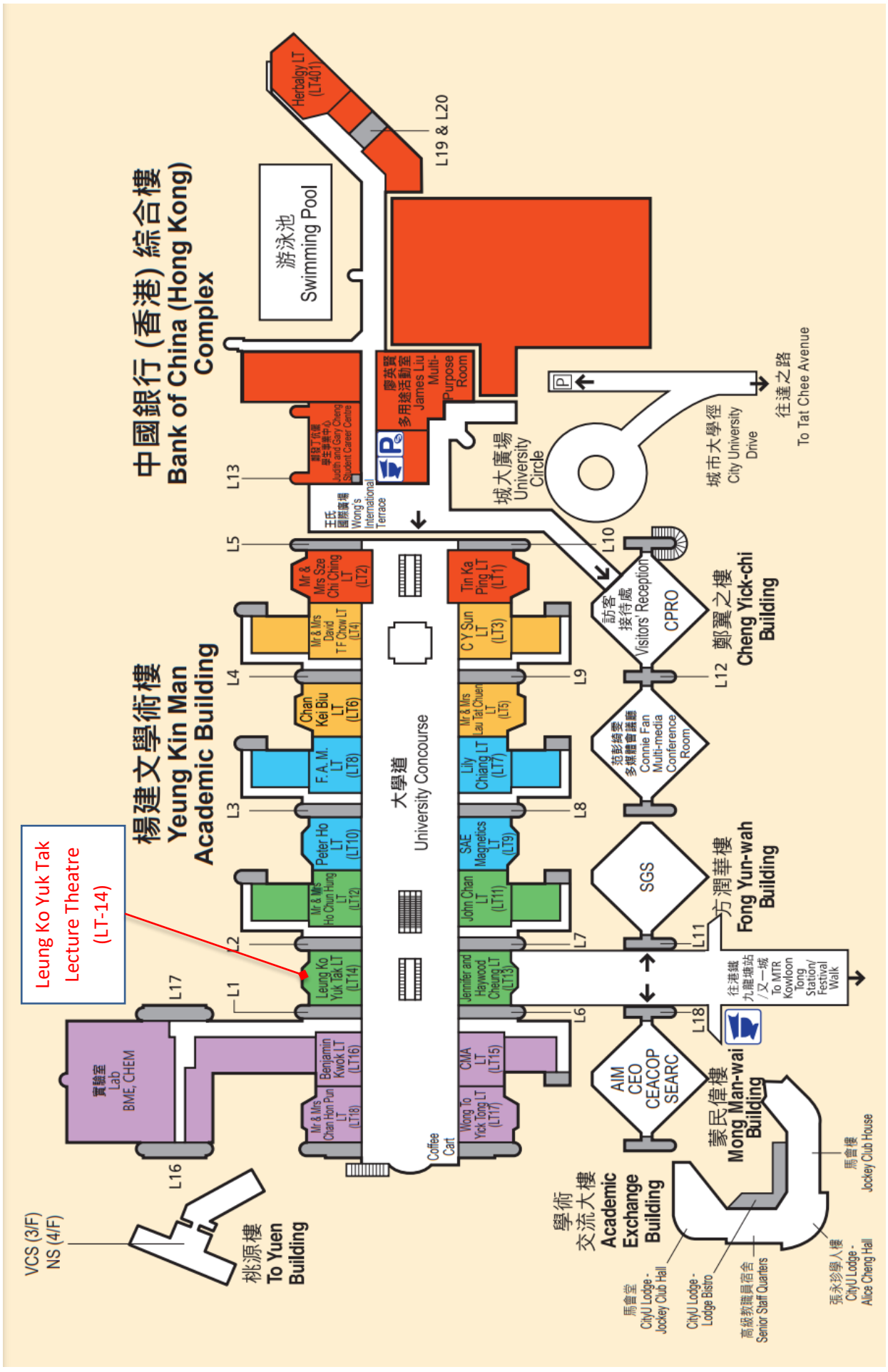
Hotels

Harbour Plaza Metropolis:	+852 3160 6807
Eaton, Hong Kong:	+852 2710 1828
Royal Park Hotel:	+852 2695 9291
InterContinental Grand Stanford Hong Kong:	+852 2731 2882
Royal Plaza Hotel:	+852 2622 6290

MISC

Immigration Department:	+852 2824 6111
Hong Kong Police:	999

CityU Location Plan



Program at a Glance

Time	3 June 2024, Monday
9:00 - 9:30	Registration
9:30 – 9:40	LT-14 Opening Ceremony
	(Chair: Ya Yan LU)
9:40 – 10:30	**PLENARY I: Dany LEVIATAN**
	Photo Session
10:30 – 11:00	Coffee Break
	LT-14 (Chair: Xin LI)
11:00 – 11:30	I-1: Andrei MARTINEZ-FINKELSHTEIN
11:35 – 12:05	I-2: Howard COHL
12:05 – 14:00	Lunch Break
	LT-14 (Chair: Peter MILLER)
14:00 – 14:50	**PLENARY II: Chun LIU**
14:55 – 15:25	I-3: Meng YANG
15:30 – 16:00	I-4: Yu-Tian LI
16:00 – 16:30	Coffee Break
	CityU Lodge Academic Exchange Building, City University of Hong Kong
18:30 – 21:00	**Welcome Reception**
LT-14: Lecture 14, Level 4, Yeung Kin Man Academic Building (Academic 1)	

Program at a Glance

Time	4 June 2024, Tuesday
8:30 – 9:00	Registration
	LT-14 (Chair: Andrei MARTINEZ-FINKELSHTEIN)
9:00 – 9:50	**PLENARY III: Walter VAN ASSCHE**
9:55 – 10:45	**PLENARY IV: Nalini JOSHI**
10:45 – 11:15	Coffee Break
	LT-14 (Chair: Walter VAN ASSCHE)
11:15 – 11:45	I-5: Xin LI
11:50 – 12:20	I-6: Yu LIN
12:20 – 14:00	Lunch Break
	LT-14 (Chair: Dan DAI)
14:00 – 14:50	**PLENARY V: Adri OLDE DAALHUIS**
14:55 – 15:25	I-7: Gergo NEMES
15:30 – 16:00	I-8: Xiang-Sheng WANG
16:00 – 16:30	Coffee Break
	LT-14 (Chair: Dan DAI)
16:30 – 16:50	C-1: Alexander GUTERMAN
16:50 – 17:10	C-2: Hao-Ning WU
17:10 – 17:30	C-3: Xinrong MA
17:30 – 17:50	C-4: Mika TANDA

Program at a Glance

Time	5 June 2024, Wednesday
8:30 – 9:00	Registration
	LT-14 (Chair: Lun ZHANG)
9:00 – 9:50	**PLENARY VI: Juncheng WEI**
9:55 – 10:45	**PLENARY VII: Peter MILLER**
10:45 – 11:15	Coffee Break
	LT-14 (Chair: Adri OLDE DAALHUIS)
11:15 – 11:45	I-9: Sheehan OLVER
11:50 – 12:20	I-10: Xiang-Ke CHANG
12:20 – 14:00	Lunch Break
	LT-14 (Chair: Juncheng WEI)
14:00 – 14:50	**PLENARY VIII: Michael WARD**
14:55 – 15:25	I-11: Yu-Qiu ZHAO
15:30 – 16:00	I-12: Zhiguo LIU
16:00 – 16:30	Coffee Break
	LT-14 (Chair: Yu-Qiu ZHAO)
16:30 – 17:00	I-13: Shuai-Xia XU
17:05 – 17:35	I-14: Zhi-Tao WEN
	City Chinese Restaurant 8/F, Bank of China (Hong Kong) Complex, City University of Hong Kong
18:30 – 21:00	**Banquet**

Program at a Glance

Time	6 June 2024, Thursday
8:30 – 9:00	Registration
	LT-14 (Chair: Dingxuan ZHOU)
9:00 – 9:50	**PLENARY IX: Yang WANG**
9:55 – 10:25	I-15: Eric GALAPON
10:30 – 11:00	Coffee Break
	LT-14 (Chair: Xiang-Sheng WANG)
11:00 – 11:20	C-5: Hiroyuki CHIHARA
11:20 – 11:40	C-6: Christian TICA
11:40 – 12:00	C-7: Kouichi TAKEMURA
12:00 – 14:00	Lunch Break
	LT-14 (Chair: Chunhua OU)
14:00 – 14:50	**PLENARY X: Huaxiong HUANG**
14:55 – 15:25	I-16: Hongying SHU
15:30 – 16:00	I-17: Zhenghui FENG
16:00 – 16:30	Coffee Break
	LT-14 (Chair: Gergo NEMES)
16:30 – 16:50	C-8: Reynaldo YLANAN
16:50 – 17:10	C-9: Hideshi YAMANE

ICAA 2024, Day 1, 3 June 2024, Monday

HK Time		Venue
9:00 - 9:30	**Registration opens**	LT-14
9:30 – 9:40	**Opening Ceremony**	
9:40 – 10:30	**PLENARY TALK I: Shape preserving approximation of periodic functions** *Prof. Dany Leviatan, Tel Aviv University, Israel* Session Chair: Prof. Ya Yan Lu	
	Photo Session	
10:30 – 11:00	**Coffee Break**	
11:00 – 11:30	**INVITED TALK 1: On the flow of zeros of derivatives of polynomials** *Andrei Martinez-Finkelshtein, Baylor University (USA) and University of Almeria (Spain)* Session Chair: Prof. Xin Li	LT-14
11:35 – 12:05	**INVITED TALK 2: On the flow of zeros of derivatives of polynomials** *Howard S. Cohl, National Institute of Standards and Technology, USA* Session Chair: Prof. Xin Li	
12:05 – 14:00	**Lunch Break**	
14:00 – 14:50	**PLENARY TALK II: Active materials and reactive fluids** *Prof. Chun Liu, Illinois Institute of Technology, USA* Session Chair: Prof. Peter Miller	LT-14
14:55 – 15:25	**INVITED TALK 3: 2D Coulomb Gases and Partition Functions** *Meng Yang, Great Bay University, China* Session Chair: Prof. Peter Miller	
15:30 – 16:00	**INVITED TALK 4: Asymptotic analysis and connection formulas for the first Painleve equation** *Yu-Tian Li, Chinese University of Hong Kong, China* Session Chair: Prof. Peter Miller	
16:00 – 16:30	**Coffee Break**	
18:30 – 21:00	**Welcome Reception**	CityU Lodge

ICAA 2024, Day 2, 4 June 2024, Tuesday

HK Time		Venue
8:30 – 9:00	**Registration opens**	LT-14
9:00 – 9:50	**PLENARY TALK III: Asymptotics for (multiple) little q-Jacobi polynomials** *Prof. Walter Van Assche, Katholieke Universiteit Leuven, Belgium* Session Chair: Prof. Andrei Martinez-Finkelshtein	
9:55 – 10:45	**PLENARY TALK IV: Motion, monodromy and asymptotics** *Prof. Nalini Joshi, The University of Sydney, Australia* Session Chair: Prof. Andrei Martinez-Finkelshtein	
10:45 – 11:15	**Coffee Break**	
11:15 – 11:45	**INVITED TALK 5: On n-th Order Asymptotics of Bernstein Polynomials and Their Derivatives** *Xin Li, University of Central Florida, USA* Session Chair: Prof. Walter Van Assche	LT-14
11:50 – 12:20	**INVITED TALK 6: Asymptotics of the Charlier polynomials via difference equation methods** *Yu Lin, South China University of Technology, China* Session Chair: Prof. Walter Van Assche	
12:20 – 14:00	**Lunch Break**	
14:00 – 14:50	**PLENARY TALK V: Exponentially-improved asymptotics for q-difference equations** *Prof. Adri Olde Daalhuis, University of Edinburgh, UK* Session Chair: Prof. Dan Dai	LT-14
14:55 – 15:25	**INVITED TALK 7: Uniform asymptotic smoothing of the higher-order Stokes phenomenon** *Gergo Nemes, Tokyo Metropolitan University, Japan* Session Chair: Prof. Dan Dai	
15:30 – 16:00	**INVITED TALK 8: Error bounds for the asymptotic expansions of the Jacobi polynomials** *Xiang-Sheng Wang, University of Louisiana at Lafayette, USA* Session Chair: Prof. Dan Dai	
16:00 – 16:30	**Coffee Break**	
16:30 – 16:50	**CONTRIBUTED TALK 1: Matrix integrability and its applications** *Alexander Guterman, Bar-Ilan University, Israel* Session Chair: Prof. Dan Dai	LT-14
16:50 – 17:10	**CONTRIBUTED TALK 2: Hyperinterpolation, Marcinkiewicz-Zygmund property, and their use for spectral methods** *Hao-Ning Wu, The University of Hong Kong, Hong Kong* Session Chair: Prof. Dan Dai	
17:10 – 17:30	**CONTRIBUTED TALK 3: A general q-series transformation and its applications to**	

	multi Rogers-Ramanujan-Slater identities *Xinrong Ma, Soochow University, China* Session Chair: Prof. Dan Dai
17:30 – 17:50	**CONTRIBUTED TALK 4: The asymptotic expansions of the standard solutions of the hypergeometric differential equation with a simple-pole-type turning point with respect to a parameter** *Mika Tanda, Otemon Gakuin University, Japan* Session Chair: Prof. Dan Dai

ICAA 2024, Day 3, 5 June 2024, Wednesday

HK Time		Venue
8:30 – 9:00	**Registration opens**	LT-14
9:00 – 9:50	**PLENARY TALK VI: Optimal Beckner's Inequality for axially symmetric functions on S4; S6 and S8** *Prof. Juncheng Wei, University of British Columbia, Canada* Session Chair: Prof. Lun Zhang	
9:55 – 10:45	**PLENARY TALK VII: Universality in the Small-Dispersion Limit of the Benjamin Ono Equation** *Prof. Peter D. Miller, University of Michigan, USA* Session Chair: Prof. Lun Zhang	
10:45 – 11:15	**Coffee Break**	
11:15 – 11:45	**INVITED TALK 9: Equilibrium measures for attractive-repulsive power law interactions** *Sheehan Olver, Imperial College London, UK* Session Chair: Prof. Adri Olde Daalhuis	LT-14
11:50 – 12:20	**INVITED TALK 10: Forward/inverse spectral analysis related to peakon flows** *Xiang-Ke Chang, Chinese Academy of Sciences, China* Session Chair: Prof. Adri Olde Daalhuis	
12:20 – 14:00	**Lunch Break**	
14:00 – 14:50	**PLENARY TALK VIII: Symmetry-Breaking Patterns with Equal Diffusivities and Diffusion-Induced Synchrony for Compartmental Reaction-Diffusion Systems** *Prof. Michael J. Ward, University of British Columbia, Canada* Session Chair: Prof. Juncheng Wei	LT-14
14:55 – 15:25	**INVITED TALK 11: Real solutions of the fourth Painleve equation: Asymptotics and applications** *Yu-Qiu Zhao, Sun Yat-sen University, China* Session Chair: Prof. Michael J. Ward	
15:30 – 16:00	**INVITED TALK 12: A new q-partial differential equation** *Zhiguo Liu, East China Normal University, China* Session Chair: Prof. Michael J. Ward	
16:00 – 16:30	**Coffee Break**	
16:30 – 17:00	**INVITED TALK 13: Asymptotics of the finite-temperature sine kernel determinant** *Shuai-Xia Xu, Sun Yat-sen University, China* Session Chair: Prof. Yu-Qiu Zhao	LT-14
17:05 – 17:35	**INVITED TALK 14: The possible orders of transcendental meromorphic solutions of linear difference equations with polynomial coecients** *Zhi-Tao Wen, Shantou University, China* Session Chair: Prof. Yu-Qiu Zhao	

18:30 – 21:00	**Banquet**	City Chinese Restaurant

ICAA 2024, Day 4, 6 June 2024, Thursday

HK Time		Venue
8:30 – 9:00	**Registration opens**	LT-14
9:00 – 9:50	**PLENARY TALK IX: Generative Adversarial Nets (GAN)** *Prof. Yang Wang, The Hong Kong University of Science and Technology, Hong Kong* Session Chair: Prof. Dingxuan Zhou	
9:55 – 10:25	**INVITED TALK 15: Revisiting the problem of missing terms: From Wong's distributional approach to asymptotics to nite-part integration** *Eric A. Galapon, University of the Philippines, Philippines* Session Chair: Prof. Dingxuan Zhou	
10:30 – 11:00	**Coffee Break**	
11:00 – 11:20	**CONTRIBUTED TALK 5: Geodesic X-ray transform and streaking artifacts on simple surfaces or on spaces of constant curvature** *Hiroyuki Chihara, University of the Ryukyus, Japan* Session Chair: Prof. Xiang-Sheng Wang	LT-14
11:20 – 11:40	**CONTRIBUTED TALK 6: Resummation and Extrapolation of Divergent Series by Finite-Part Integration** *Christian Tica, University of the Philippines, Philippines* Session Chair: Prof. Xiang-Sheng Wang	
11:40 – 12:00	**CONTRIBUTED TALK 7: Kernel function, q-integral transformation and q - Heunequations** *Kouichi Takemura. Ochanomizu University, Tokyo, Japan* Session Chair: Prof. Xiang-Sheng Wang	
12:00 – 14:00	**Lunch Break**	
14:00 – 14:50	**PLENARY TALK X: Water and Ion Transport in the Optic Nerve: The Role of Glial Cells** *Prof. Huaxiong Huang, Beijing Normal University, China* Session Chair: Prof. Chunhua Ou	LT-14
14:55 – 15:25	**INVITED TALK 16: Spatiotemporal patterns of a structured spruce bud-worm diffusive model** *Hongying Shu, Shaanxi Normal University, China* Session Chair: Prof. Chunhua Ou	
15:30 – 16:00	**INVITED TALK 17: Component Selection and Variable Selection for General Robust Mixture Regression Models** *Zhenghui Feng, Harbin Institute of Technology, China* Session Chair: Prof. Chunhua Ou	
16:00 – 16:30	**Coffee Break**	
16:30 – 16:50	**CONTRIBUTED TALK 8: Regularized Limit and Contour Integral Representation**	LT-14

	of Finite-Part Integral *Reynaldo P. Ylanan, University of the Philippines, Philippines* Session Chair: Prof. Gergo Nemes
16:50-17:10	**CONTRIBUTED TALK 9: Asymptotic expansions of finite Hankel transforms and the surjectivity of convolution operator** *Hideshi Yaman, Kwansei Gakuin University, Japan* Session Chair: Prof. Gergo Nemes

Abstracts

International Conference on Analysis and Applications

in honor of Roderick S. C. Wong's 80th birthday

3 June - 6 June, 2024
City University of Hong Kong

Co-organized by
Liu Bie Ju Centre for Mathematical Sciences
Department of Mathematics

Contents

1 Plenary Talks

Water and Ion Transport in the Optic Nerve: The Role of Glial Cells

Huaxiong Huang
Research Center for Mathematics, Beijing Normal University, China
Email: hhuang@uic.edu.cn

In recent years, there has been a growing interest in understanding the mechanisms of water transportation in the brain and the central nervous system (CNS) due to its potential implications for neurodegenerative diseases. The optic nerve, a component of the CNS, offers an ideal starting point for investigation due to its relatively simple geometry and structure, along with its similarities to other CNS regions.

In this presentation, we provide an overview of the critical role played by glial cells in maintaining homeostasis through the regulation of water and ion transport in the optic nerve. Additionally, we introduce a multi-compartment mathematical model and present numerical solutions under physiologically realistic conditions.

This collaborative work involves contributions from Shixin Xu, Shanfeng Xiao, Y. Zhu, and Bob Eisenberg.

Motion, monodromy and asymptotics

Nalini Joshi
School of Mathematics and Statistics, The University of Sydney, Australia
Email: nalini.joshi@sydney.edu.au

Felix Klein said that the study of new transcendental functions defined by differential equations was "the central problem of the whole of modern mathematics". The same impetus drove the search that led to differential equations called the Painlevé equations, more than a century ago. The subtle properties of their solutions are still being discovered and new applications are growing in physics. The solutions of the discrete Painlevé equations show tantalizingly similar properties, but the study of these transcendental functions remains far from complete.

In this talk, I will give an overview of three major mathematical threads in this area, which starts with dynamics of planets, the study of linear differential and difference equations, and gives a glimpse of the geometry underlying

these alluring equations. At the end, I will briefly mention some new work with Pieter Roffelsen (Sydney) on the solutions of the q-discrete version of the sixth Painlevé equation.

Shape preserving approximation of periodic functions

Dany Leviatan
Tel Aviv University, Israel
Email: leviatan@tauex.tau.ac.il

We wish to approximate a continuous 2π-periodic function that changes its monotonicity or convexity finitely many times in its period, by trigonometric polynomials that follow exactly these changes, namely are nondecreasing and nonincreasing, respectively convex or concave, exactly where the function is. Especially, we investigate the validity of Jackson-type estimates in this type of approximation. This type of approximation is called comonotone, respectively coconvex approximation. There are no Jackson type estimates for the so called co-q-monotone, $q > 2$, approximation. We discuss whether these estimates depend on the disposition of the points of change, namely the extremum, respectively, the inflection points of the function to be approximated. It is interesting to point out that the results for coconvex approximation are similar to those we know for coconvex algebraic polynomials approximation of a continuous function on a finite interval, while the results for comonotone approximation are substantially different than the analogous results for comonotone algebraic polynomials approximation of a continuous function on a finite interval.

Let $f \in C$ be a 2π-periodic function that changes its monotonicity, respectively convexity $2s$-times, $s \geq 1$, in a period at points Y_s. We wish to approximate f by trigonometric polynomials T_n of degree $< n$, which follow its changes in monotonicity, respectively convexity. For $l = 1, 2$, we denote by

$$E_n^{(l)}(f, Y_s) = \inf \|f - T_n\|,$$

where the infimum is taken on all such T_n, the degree of such approximation.

For $f \in C^r$, $r \geq 0$, we are interested in estimates of the form

$$E_n^{(l)}(f, Y_s) \leq \frac{c(r, k, s)}{n^r} \omega_k(f^{(r)}, 1/n), \quad n \geq N,$$

where ω_k is the kth modulus of smoothness of f.

In general, such estimates exist with $N = N(r, k, Y_s)$. We will determine for which triplets (r, k, s), we may take $N = 1$.

Active materials and reactive fluids

Chun Liu
Department of Applied Mathematics, Illinois Institute of Technology, USA
Email: cliu124@iit.edu

In this talk I will present a general theory for active fluids which convert chemical energy into various type of mechanical energy. This is the extension of the classical energetic variational approaches for mechanical systems. The methods will cover a wide range of both chemical reaction kinetics and mechanical processes. This is a joint project with many collaborators, in particular, Bob Eisenberg, Yiwei Wang and Tengfei Zhang.

Universality in the Small-Dispersion Limit of the Benjamin-Ono Equation

Peter D. Miller
Department of Mathematics, University of Michigan, USA
Email: millerpd@umich.edu

This talk concerns the Benjamin-Ono (BO) equation of internal wave theory, and properties of the solution of the Cauchy initial-value problem in the situation that the initial data is fixed but the coefficient of the nonlocal dispersive term in the equation is allowed to tend to zero (i.e., the zero-dispersion limit). It is well-known that existence of a limit requires the weak topology because high-frequency oscillations appear even though they are not present in the initial data. Physically, this phenomenon corresponds to the generation of a dispersive shock wave. In the setting of the Korteweg-de Vries (KdV) equation, it has been shown that dispersive shock waves exhibit a universal form independent of initial data near the two edges of the dispersive shock wave, and also near the gradient catastrophe point for the inviscid Burgers equation from which the shock wave forms. In this talk, we will present corresponding universality results for the BO equation. These have quite a different character than in the KdV case; while for KdV one has universal wave profiles expressed in terms of solutions of Painlevé-type equations, for BO one instead has expressions in terms of classical Airy functions

and Pearcey integrals. These results are proved for general rational initial data using a new approach based on an explicit formula for the solution of the Cauchy problem for BO. This is joint work with Elliot Blackstone, Louise Gassot, Patrick Gérard, and Matthew Mitchell.

Exponentially-improved asymptotics for q-difference equations: ${}_2\phi_0$ and $q\mathrm{P_I}$

Nalini Joshi, Adri Olde Daalhuis*
School of Mathematics, University of Edinburgh, Edinburgh, UK
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Usually when solving differential or difference equations via series solutions one encounters divergent series in which the coefficients grow like a factorial. Surprisingly, in the q-world the nth coefficient is often of the size $q^{-\frac{1}{2}n(n-1)}$, in which $q \in (0,1)$ is fixed. Hence, the divergence is much stronger, and one has to introduce alternative Borel and Laplace transforms to make sense of these formal series. We will discuss exponentially-improved asymptotics for the basic hypergeometric function ${}_2\phi_0$ and for solutions of the q-difference first Painlevé equation $q\mathrm{P_I}$. These are optimal truncated expansions, and re-expansions in terms of new q-hyperterminant functions. The re-expansions do incorporate the Stokes phenomena.

Asymptotics for (multiple) little q-Jacobi polynomials

Walter Van Assche
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Little q-Jacobi polynomials and multiple little q-Jacobi polynomials satisfy orthogonality conditions with respect to a discrete measure on the q-lattice $\{q^k, k = 0, 1, 2, \ldots\}$ with $0 < q < 1$. We will give the asymptotics of the zeros of $p_n(x^n)$ for these polynomials which shows that these zeros are dense on an annulus. We formulate the q-Riemann-Hilbert problem for the multiple little q-Jacobi polynomials, which for the orthogonal polynomials was introduced by Joshi and Latimer in 2021. We then use this q-RHP to deduce asymptotic properties of these polynomials.

Generative Adversarial Nets (GAN)

Yang Wang
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Generative Adversarial Nets (GAN) have been one of the most exciting developments in machine learning and AI. In this talk I will give an introduction to GAN, and discuss the difficulty of training a GAN. I'll propose a framework to learn deep generative models via Variational Gradient Flow (Vgrow) on probability measure spaces. Connections of our proposed VGrow method with other popular methods, such as VAE, GAN and flow-based methods, have been established in this framework, gaining new insights of deep generative learning.

Symmetry-Breaking Patterns with Equal Diffusivities and Diffusion-Induced Synchrony for Compartmental Reaction-Diffusion Systems

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We investigate pattern formation for a 2D PDE-ODE bulk-cell model, where one or more bulk diffusing species are coupled to nonlinear intracellular reactions that are confined within a disjoint collection of small circular compartments. The bulk species are coupled to the spatially segregated intracellular reactions through Robin conditions across the cell boundaries. For this compartmental-reaction diffusion system, we show that symmetry-breaking bifurcations leading to stable asymmetric steady-state patterns, as regulated by a membrane binding rate ratio, occur even when two bulk species have equal bulk diffusivities. This result is in distinct contrast to the usual, and often biologically unrealistic, large differential diffusivity ratio requirement for Turing pattern formation from a spatially uniform state. Secondly, for the case of one-bulk diffusing species in $\mathbb{R}^2$, we derive a new memory-dependent ODE integro-differential system that characterizes how intracellular oscillations in the collection of cells are coupled through the PDE bulk-diffusion field. By using a fast numerical approach relying on

the "sum-of-exponentials" method to derive a time-marching scheme for this nonlocal system, diffusion induced synchrony is examined for various spatial arrangements of cells. This theoretical modeling framework, relevant when spatially localized nonlinear oscillators are coupled through a PDE diffusion field, is distinct from the traditional Kuramoto paradigm for studying oscillator synchronization on networks or graphs. Numerical challenges, some still only partially resolved, for numerically implementing our analytical theory are discussed. (Joint work with Merlin Pelz, UBC).

Optimal Beckner's Inequality for axially symmetric functions on S^4, S^6 and S^8

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We prove that axially symmetric solutions to the Q-curvature type problem

$$\alpha P_{2n} u + n! \left(1 - \frac{e^{2nu}}{\int_{\mathbb{S}^{2n}} e^{2nu}}\right) = 0 \qquad \text{on } \mathbb{S}^{2n}$$

must be constants, provided that $\frac{1}{2} \leq \alpha < 1$ and $2n = 4, 6, 8$. In view of the existence of non-constant solutions obtained by Gui-Hu-Xie (2022) for $\frac{1}{7} < \alpha < \frac{1}{2}$, this result is sharp. This result closes the gap of the related results in Gui-Hu-Xie (2022) which proved a similar uniqueness result for $\alpha \geq 0.6168$ (when $2n = 6$) and $\alpha \geq 0.867$ (when $2n = 8$). The improvement is based on two types of new estimates: one is a better estimate of the semi-norm $\lfloor G \rfloor^2$, the other one is a family of refined estimates on Gegenbauer coefficients, such as large parameters asymptotic expansions for Gegenbauer polynomials proved in Nemes and Olde Daalhuis (2020) and some cancelling properties. (Joint work with C. Gui, T. Li and Z. Ye.)

2 Invited Talks

Forward/inverse spectral analysis related to peakon flows

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A family of integrable PDEs admit the so-called peakon solutions, the dynamics of which may be characterized by implementing the related forward/inverse spectral analysis. The spectral problems often involve non-self-adjoint operators, while the inverse spectral analysis usually involve Hermite-Padé approximation problems, which have motivated new (bi)orthogonality and random matrix models, etc. This talk will focus on the peakon flows of a two-component Novikov equation with a non-self-adjoint 4×4 Lax operator and the related forward/inverse spectral analysis. Differences and challenges are highlighted in comparison to the previous works.

Orthogonality relations for the q and q^{-1}-symmetric and dual polynomials in the q-Askey scheme

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In this talk I will summarize the current state of known orthogonality relations for the q and q^{-1}-symmetric and dual subfamilies of the Askey–Wilson polynomials in the q-Askey scheme. These polynomials are the continuous dual q and q^{-1}-Hahn polynomials, the q and q^{-1}-Al-Salam–Chihara polynomials, the continuous big q and q^{-1}-Hermite polynomials and the continuous q and q^{-1}-Hermite polynomials and their dual counterparts which are connected with the big q-Jacobi polynomials, the little q-Jacobi polynomials and the q and q^{-1}-Bessel polynomials. The q^{-1}-symmetric polynomials in the q-Askey scheme satisfy an indeterminate moment problem, so they satisfy an infinite number of orthogonality relations for these polynomials. Among the infinite number of orthogonality relations for the q^{-1}-symmetric families, we attempt to summarize those currently known. These fall into several classes,

including continuous orthogonality relations and infinite discrete (including bilateral) orthogonality relations. Using symmetric limits, we derive a new infinite discrete orthogonality relation for the continuous big q^{-1}-Hermite polynomials. Using duality relations, we explore orthogonality relations for and from the dual families associated with the q and q^{-1}-symmetric subfamilies of the Askey–Wilson polynomials. In order to describe convergence properties for these polynomials, we perform large degree asymptotics using the Darboux method. In order to apply the Darboux method, we derive a generating function with two free parameters for the q^{-1}-Al-Salam–Chihara polynomials which has natural limits to the lower q^{-1}-symmetric families.

Component Selection and Variable Selection for General Robust Mixture Regression Models

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Our work focuses on model selection and variable selection in finite mixture regression models, which are commonly used to account for heterogeneity in populations and situations where standard regression model assumptions may not hold. To expand the range of applicable distributions beyond the Gaussian distribution, we explore other distributions such as the exponential power distribution, the skew-normal distribution, and so on. To enable simultaneous model estimation, order selection, and variable selection, we propose a penalized likelihood method that imposes penalties on both mixing proportions and regression coefficients, which we call the "double-penalized likelihood method" (DPL). We study four double-penalized likelihood functions and compare their performances, as well as investigate the consistency of estimators, order selection, and variable selection. We propose a modified EM algorithm to implement the DPL method. Numerical simulations demonstrate the effectiveness of our proposed method and algorithm, and we provide a real data analysis to illustrate the application of our approach. Estimation and order selection for univariate and multivariate exponential power mixture models are also presented as examples. Overall, our study contributes to the development of mixture regression models and provides a useful tool for model and variable selection.

Revisiting the problem of missing terms: From Wong's distributional approach to asymptotics to finite-part integration

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In this talk we revisit the problem of missing terms that arises from term-by-term integration leading to an infinite series of divergent integrals. We discuss the problem in the context of the asymptotic and exact evaluation of the Stieltjes transform in the large and small regimes. The problem is first elucidated in the large regime by Wong's solution to the missing terms by means of the distributional interpretation of the divergent integrals arising from term-by-term integration. The problem is next considered in the small regime by the recent method of finite-part-integration where the divergent integrals are assigned values equal to their finite-parts. The relationship between the two approaches is then established. Finally, recent advances in the theory and applications of finite-part integration are discussed.

Asymptotic analysis and connection formulas for the first Painlevé equation

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We consider a connection problem of the first Painlevé equation (PI), trying to connect the local behavior (Laurent series) near poles and the asymptotic behavior as the variable t tends to the negative infinity for real PI functions. We get a classification of the real PI functions in terms of (p, H) so that they behave differently at the negative infinity, where p is the location of a pole and H is the free parameter in the Laurent series. Some limiting-form connection formulas of PI functions are obtained for large H. Specifically, for the real tritronquée solution, the large-n asymptotic formulas of p_n and H_n are obtained, where p_n is the n-th pole on the real line in the ascending order and H_n is the associated free parameter. Our approach is based on the

complex WKB method (also known as the method of uniform asymptotics) introduced by Bassom et al. in their study on the connection problem of the second Painlevé transcendent [Arch. Ration. Mech. Anal., 143 (1998), pp. 241–271]. Several numerical simulations are carried out to verify our main results. Meanwhile, we obtain the phase diagram of PI solutions in the (p, H) plane, which somewhat resembles the Brillouin zones in solid-state physics. Our asymptotic and numerical results partially answer Clarkson's open question on the connection problem of the first Painlevé transcendent.

On n-th Order Asymptotics of Bernstein Polynomials and Their Derivatives

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We study the n-th order asymptotics of Bernstein polynomials and their derivatives for functions whose n-th derivatives are discontinuous. In addition to the classical tools, a key auxiliary result in the asymptotic behavior of the one-sided moments of the discrete Gaussian distribution will play an important role in establishing our results.

Asymptotics of the Charlier polynomials via difference equation methods

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In this talk, we will present uniform and non-uniform asymptotics of the Charlier polynomials using only difference equation methods. The Charlier polynomials are special because they do not fit into the framework of the turning point theory, even though they play a crucial role in the Askey scheme. Asymptotic approximations are obtained for the outside region, an intermediate region, and near the turning points, respectively. In particular, we obtain uniform asymptotic approximation at a pair of coalescing turning

points with the aid of a local transformation. This is joint work with Xiaomin Huang and Yuqiu Zhao.

A new q-partial differential equation

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A q-partial differential equation is a q-analogue of a partial differential equation. A few q-partial differential equations have been studied in the past decade. It turns out that q-partial differential equations are quite useful in deriving q-formulas. In this talk, I will introduce a new q-partial differential equation and discuss its application in q-series.

On the flow of zeros of derivatives of polynomials

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Assume we have a sequence of polynomials whose asymptotic zero distribution is known. What can be said about the zeros of their derivatives? Especially if we differentiate each polynomial several times, proportional to their degree?

This simple-to-formulate problem has recently attracted the attention of several researchers. Both the problem and the methods of its solution have exciting connections with free probability, random matrices, and approximation theory on the complex plane. This is a partial survey in which I will explain some known results in this direction and our approach to the problem. This is a joint work in progress with E. Rakhmanov from the University of South Florida.

Uniform asymptotic smoothing of the higher-order Stokes phenomenon

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The Stokes phenomenon refers to the apparent discontinuous change in the form of a function's asymptotic expansion across certain rays in the complex plane, known as Stokes lines. This phenomenon was first observed by G. G. Stokes while studying the asymptotic behaviour of the Airy function. R. B. Dingle proposed a set of rules for locating Stokes lines and continuing asymptotic expansions across them. One of these rules, known as the "final main rule", states that half of the discontinuity in form occurs upon reaching the Stokes line, and the other half occurs upon leaving it. M. V. Berry demonstrated that if an asymptotic expansion is terminated just before its numerically least term, the transition between two different asymptotic forms across a Stokes line is smooth rather than discontinuous, as conventionally interpreted. In accordance with Dingle's final main rule, Berry's law predicts a multiplier of $\frac{1}{2}$ for the emerging small exponentials on a Stokes line. In this talk, we shall consider asymptotic expansions in which the multipliers of exponentially small contributions may no longer adhere to Dingle's rule: their values can deviate from $\frac{1}{2}$ along a Stokes line, and they can be non-zero solely on the line itself. This unusual behaviour of the multipliers results from higher-order Stokes phenomena. We demonstrate that these phenomena are rapid but smooth transitions in the remainder terms of optimally truncated hyperasymptotic re-expansions. Achieving this requires the derivation of uniform asymptotic approximations for multidimensional integrals featuring coalescence phenomena between saddles and poles. While the error function describes the smooth transition of an ordinary Stokes phenomenon, the smoothing of the higher-order Stokes phenomenon is governed by a more general transcendental function. Part of this research is joint work with C. J. Howls, J. R. King and A. B. Olde Daalhuis.

Equilibrium measures for attractive-repulsive power law interactions

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We discuss numerical methods for computing equilibrium measures which are of significant importance in random matrix theory, and extensions to the case of attractive-repulsive interactions, in particular where the particles interact according to a power-law. This includes new recurrence relationships of power law kernels applied to weighted orthogonal polynomials, including in higher dimensions.

Spatiotemporal patterns of a structured spruce budworm diffusive model

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We formulate and analyze a general reaction-diffusion equation with delay, inspired by age-structured spruce budworm population dynamics with spatial diffusion by matured individuals. The model has its particular feature for bistability due to the incorporation of a nonlinear birth function (Ricker's function) and a Holling type function of predation by birds. Here we establish some results about the global dynamics, in particular, the stability of and global Hopf bifurcation from the spatially homogeneous steady state when the maturation delay is taken as a bifurcation parameter. We also use a degree theoretical argument to identify intervals for the diffusion rate when the model system has a spatially heterogeneous steady state. Numerical experiments presented show interesting spatialtemporal patterns.

Error bounds for the asymptotic expansions of the Jacobi polynomials

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We derive explicit error bounds for the asymptotic expansions of the Jacobi polynomials as the polynomial degree approaches infinity. A new technique is developed to address the difficulties arising from the logarithmic singularity of the phase function in the integral representation of the Jacobi polynomials.

The possible orders of transcendental meromorphic solutions of linear difference equations with polynomial coefficients

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We consider possible orders of transcendental meromorphic solutions of linear difference equations

$$P_m(z)\Delta^m f(z) + \cdots + P_1(z)\Delta f(z) + P_0(z)f(z) = 0, \qquad (+)$$

where $P_j(z)$ are polynomials for $j = 0, \ldots, m$. Firstly, we give the condition on existence of transcendental entire solutions of order less than 1 of difference equations (+). Secondly, we give a list of all possible orders which are less than 1 of transcendental entire solutions of difference equations (+). Moreover, the maximum number of distinct orders which are less than 1 of transcendental entire solutions of difference equations (+) are shown. Further, in both two cases, for a given difference equation (+) with polynomial coefficients, we can construct a meromorphic solution of (+) of order $\rho(f) = \rho$ for any $\rho \in [1, +\infty)$. Thirdly, for any given rational number $0 < \rho < 1$, we can construct a linear difference equation with polynomial coefficients which has a transcendental entire solution of order ρ. At least, some examples are illustrated for our main theorems. This is a joint work with Katsuya Ishizaki.

Asymptotics of the finite-temperature sine kernel determinant

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In the present paper, we study the asymptotics of the Fredholm determinant $D(x,s)$ of the finite-temperature deformation of the sine kernel. The determinant represents the probability that there is no particles on the interval $(-x/\pi, x/\pi)$ in the bulk scaling limit of the finite-temperature fermion system. The variable s in $D(x,s)$ is related to the temperature. The determinant also corresponds to the finite-temperature correlation function of one dimensional Bose gas. We derive the asymptotics of $D(x,s)$ in several different regions in the (x,s)-plane. A third order phase transition is observed in the asymptotic expansions for both x and s tend to infinity at certain related speed. The phase transition is then shown to be described by the integral involving the Hastings-McLeod solution of the second Painlevé equation.

2D Coulomb Gases and Partition Functions

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We consider 2D Coulomb gases with the external potential $Q(z) = |z|^2 - 2c\log|z-a|$, where $c > 0$ and $a \in \mathbb{C}$. Equivalently, this model can be realised as N eigenvalues of the complex Ginibre matrix of size $(c+1)N \times (c+1)N$ conditioned to have deterministic eigenvalue a with multiplicity cN. Depending on the values of c and a, the droplet reveals a phase transition: it is doubly connected in the post-critical regime and simply connected in the pre-critical regime. In both regimes, we derive precise large-N expansions of the free energy up to the $O(1)$ term, providing a non-radially symmetric example that confirms the Zabrodin-Wiegmann conjecture made for general planar Coulomb gas ensembles. As a consequence, our results provide asymptotic behaviours of moments of the characteristic polynomial of the complex Ginibre matrix, where the powers are of order $O(N)$. Furthermore, by combining with a duality formula, we obtain precise large deviation probabilities of the smallest eigenvalue of the Laguerre unitary ensemble. This talk is based on

the joint work with Sung-Soo Byun and Seong-Mi Seo.

Real solutions of the fourth Painlevé equation: Asymptotics and applications

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The Clarkson-McLeod solutions that exponentially decay at positive infinity furnish a family of real solutions to a special case of the fourth Painlevé equation. Using the Deift-Zhou nonlinear steepest descent method, we derive the asymptotic behaviors for these solutions at the negative infinity. This completes a proof of the conjecture of Clarkson and McLeod. As applications, we approximate the Fredholm determinant, the total integrals of the solutions, and further obtain the asymptotic approximations and a determinantal representation of the σ-form of these solutions. I will also report some progress made on the real solutions of the general fourth Painlevé equation. This talk is based on joint work with Jun XIA and Shuai-Xia XU.

3 Contributed Talks

Geodesic X-ray transform and streaking artifacts on simple surfaces or on spaces of constant curvature

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The X-ray transform on the plane or on the three-dimensional Euclidean space can be considered as the measurements of CT scanners for normal human tissue. If the human body contains metal regions such as dental implants, stents in blood vessels, metal bones, etc., the beam-hardening effect for the energy level of the X-ray causes streaking artifacts in its CT image. More precisely, if there are two strictly convex metal regions contained in the cross-section of normal human tissue, then streaking artifacts occur along the common tangent lines of the two regions.

This talk discusses this phenomenon for the geodesic X-ray transform on nontrapping simple compact Riemannian manifolds with strictly convex boundaries. We show that the streaking artifacts result from the propagation of conormal singularities on the boundary of metal regions along the common tangent geodesics under the strong and seemingly curious assumption that the manifolds are two dimensional or spaces of constant curvature. This condition ensures that every Jacobi field takes the form of the product of a scalar function and parallel transport along the geodesic. Our results clarify the geometric meaning of the theory, which was imperceptible in the known results on the Euclidean space.

Matrix integrability and its applications

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The notion of matrix integrability was introduced in [1] by Bhat and Mukherjee as a natural counterpart to the classical notion of matrix differentiation, but the question of the existence of integrable and non-integrable matrices with a given spectral properties remained open. We provide a complete answer to this question. Moreover, we establish an easy to check new criteria for matrix integrability.

To construct integrable and non-integrable matrices with a given spectral behavior we use Grothendieck dessins d'enfants that are connected embedded graphs of special structure on smooth oriented compact surfaces without boundary, see [3], and the conservative polynomials, i.e. the polynomials leaving their critical points fixed [2], arising from the theory of immediate attractive basin.

In the talk we will also discuss the relations between matrix integrability and several conjectures concerning the polynomial root distribution. In particular, we discuss the inverse Schoenberg-like inequalities following from this.

[1] B.V.R. Bhat, M. Mukherjee, Integrators of matrices, Linear Algebra Appl. 426 (2007) 71-82.

[2] A. Kostrikin, Conservative polynomials, in "Stud. Algebra Tbilisi" (1984) 115-129.

[3] G. B. Shabat, V. A. Voevodsky, Drawing curves over number fields, The Grothendieck Festschrift 3 (1990) 199-227.

Joint work with Suren Danielyan, Elena Kreines, Tuen Wai Ng, and Fedor Pakovich

A general q-series transformation and its applications to multi Rogers-Ramanujan-Slater identities

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This talk is devoted to applications of the following transformation to basic hypergeometric series

$$\sum_{i,j\geq 0}\frac{A_{i-j}q^{ij}x^j}{(q;q)_i(q;q)_j}\frac{(bq^{-i},t;q)_j}{(bt;q)_j}=\sum_{i,j,k\geq 0}\frac{A_{i-j}\tau(k)x^{j+k}}{(q;q)_i(q;q)_j(q;q)_k}\frac{(tq^{i+k},b;q)_j}{(bt;q)_j},$$

where $\{A_n\}_{n\in\mathbb{Z}}$ denotes any general sequence such that the series above converge absolutely. It may be regarded as a generalization of L. Wang et al.'s result which is used to study Nahm series (cf. J. Combin. Theory, Series A, Vol. 202 (2024),105819)

$$\sum_{i,j\geqslant 0}\frac{A_{i-j}q^{\frac{j(j-1)}{2}}x^j}{(q;q)_i(q;q)_j}=(-x;q)_\infty\sum_{i,j\geqslant 0}\frac{A_{i-j}q^{\frac{j(j-1)}{2}+ij}x^j}{(q;q)_i(q;q)_j(-x;q)_j}.$$

As direct applications of our result, some concrete new transformation formulas for the ${}_{r+1}\phi_r$ series as well as multi Rogers-Ramanujan-Slater identities are presented.

Kernel function, q-integral transformation and q-Heun equations

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Heun's differential equation is a standard form of Fuchsian differential equation with four singularities $\{0, 1, t, \infty\}$, which contains an accessory parameter. The q-Heun equation is a q-difference equation written as

$$\{a_2x^2 + a_1x + a_0\}g(x/q) - \{b_2x^2 + b_1x + b_0\}g(x) + \{c_2x^2 + c_1x + c_0\}g(xq) = 0,$$

with the condition $a_2a_0c_2c_0 \neq 0$, and it tends to Heun's differential equation as $q \to 1$. It was introduced by Hahn (1971) and was rediscovered around 2016 by considering degeneration of the Ruijsenaars-van Diejen system four times. In the paper by Komori, Noumi and Shiraishi (2009), a function $\Phi(\mathbf{x}; \mathbf{y})$ is called a kernel function for the pair of operators $(\mathcal{A}_\mathbf{x}, \mathcal{B}_\mathbf{y})$, if it satisfies

$$\mathcal{A}_\mathbf{x}\Phi(\mathbf{x}; \mathbf{y}) = \mathcal{B}_\mathbf{y}\Phi(\mathbf{x}; \mathbf{y}).$$

It appears in studies of Calogero systems, Macdonald polynomials and so on.

In this talk, we find kernel functions and related identities of the q-Heun equation and its variants. We apply them to obtain q-integral transformations of solutions to the q-Heun equation and its variants. Based on arXiv:2309.09341.

The asymptotic expansions of the standard solutions of the hypergeometric differential equation with a simple-pole-type turning point with respect to a parameter

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The Gauss hypergeometric differential equation with a large parameter de-

formed to a differential equation with a simple-pole-type turning point at the origin is considered. This equation has the formal solutions which are called WKB solutions. The WKB solutions are Borel summable and the Borel sums are analytic solutions to the equation. The Borel sums of them automatically have asymptotic expansions with respect to the parameter. In this talk, using the relations between the standard solutions of the equation in the neighborhood of the origin and the Borel sums of WKB solutions, the asymptotic expansion formulas for the standard solutions of it are obtained from the viewpoint of exact WKB analysis.

Resummation and Extrapolation of Divergent Series by Finite-Part Integration

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We present a prescription based on the method of finite-part integration to carry out the resummation and constrained extrapolation of both the divergent alternating and non-alternating pertubative expansions for the ground state energy of the anharmonic oscillator with both positive and negatively coupled sextic perturbation. In both cases, finite-part integration enabled us to transform the divergent weak-coupling perturbation expansion to a convergent extrapolant that can be used to compute the ground state energy well into the strong coupling regime. The prescription also recovered the exponentially vanishing non-perturbative imaginary part of the ground state energy which gives the lifetime of the metastable states from the real perturbation coefficients.

References:
1. Tica, Christian D., and Eric A. Galapon. "Continuation of the Stieltjes series to the large regime by finite-part integration." Proceedings of the Royal Society A 479.2276 (2023): 20230098.
2. Galapon, Eric A. "The problem of missing terms in term by term integration involving divergent integrals." Proceedings of the Royal Society A: Mathematical, Physical and Engineering Sciences 473.2197 (2017): 20160567.
3. Tica, Christian, Galapon Eric, "Exact Evaluation and extrapolation of

the divergent expansion for the Heisenberg-Euler Lagrangian I: Alternating Case," https://doi.org/10.48550/arXiv.2310.08199
4. Tica, Christian, Galapon Eric, Blancas Philip Jordan, "Exact Evaluation and extrapolation of the divergent expansion for the Heisenberg-Euler Lagrangian II: Non-alternating Case," https://arxiv.org/abs/2402.14839

Hyperinterpolation, Marcinkiewicz–Zygmund property, and their use for spectral methods

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Hyperinterpolation is a quadrature-based approximation scheme for continuous functions, which can be regarded as a discrete version of the L^2 orthogonal projection. This scheme boasts an elegant L^2 convergence theory and is relatively easy to implement. However, the convergence theory heavily relies on certain degrees of quadrature exactness, limiting its applicability in multivariate domains. In this talk, we will discuss our recent progress in addressing the limitation by relaxing the quadrature exactness requirement, aided by the Marcinkiewicz–Zygmund inequality. By deriving error bounds for approximation, we are then able to use sets of points that do not necessarily exhibit quadrature exactness for hyperinterpolation, while still maintaining reasonable error rates. Furthermore, we will showcase an application of hyperinterpolation to a quadrature-based spectral method for solving the Allen–Cahn equation on spheres. Numerical examples for both schemes of approximation and the spectral method are presented.

Asymptotic expansions of finite Hankel transforms and the surjectivity of convolution operator

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A distribution with compact suport is called invertible in the sense of Ehrenpreis and Hörmander if the convolution with it induces a surjection from $\mathcal{C}^\infty(\mathbb{R}^n)$ to itself. We give sufficient conditions for radial functions to be invertible. Our analysis is based on the asymptotic expansions of finite Hankel

transforms. The dominant term may be the contribution from the origin or from the boundary of the support of the function. We combine a formula due to Roderick Wong with our own, which allows one to calculate the asymptotic expansions of finite Hankel transforms of functions with singularities at a point other than the origin.

Regularized Limit and Contour Integral Representation of Finite-Part Integral

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Finite-part integration and regularized limit are two mathematical methods that can be used to solve problems in mathematics and physics. In this paper, we elaborate on the applications of these methods. The general form of the regularized limit for a function with simple poles of order n is determined using the partition function in number theory and the reciprocal of an infinite series. Using the concept of the regularized limit, we solve a converging part of a diverging series approaching a specific pole. We include the polygamma function, powers of gamma functions, and some trigonometric functions in our examples. We generalize the formulation of the contour integral representations of finite-part integrals with logarithmic singularity. We show some applications of these contour integral representations such as their ability to provide a numerical evaluation of the finite-part integral, their ability to aid in the evaluation of the finite-part integration of non-mellin-type divergent integrals, their parts in Stieltjes transform, and some other specific physics problems.

References:
1. E. A. Galapon, "Regularized limit, analytic continuation, and finite-part integration," Analysis and Applications, vol. 21, no. 04, pp. 841–900, 2023.
2. C. D. Tica and E. A. Galapon, "Continuation of the stieltjes series to the large regime by finite-part integration," Proc. R. Soc., vol. A 479, 2023.
3. S. Ahmed, "Reciprocal of infinite series and partition function," Integral Transforms and Special Functions, vol. 22, no. 6, pp. 443–452, 2011.