Updated on 18 Feb 2019

Program at a Glance

Time	Monday 18 Feb
9:00-10:00	**Amos RON** *(Chair: Ding Xuan ZHOU)*
10:00-11:00	**Ding Xuan ZHOU** *(Chair: Dan DAI)*
11:00-11:20	Coffee Break
11:20-12:20	**David LEVIN** *(Chair: Ding Xuan ZHOU)*
12:30 -14:00	Free Lunch Time
14:00-15:00	**Dany LEVIATAN** *(Chair: Felipe CUCKER)*
15:00-16:00	**Edmund CHIANG** *(Chair: Dan DAI)*
16:00-16:20	Coffee Break
16:20- 17:20	**Haim Avron** *(Chair: Xiaosheng ZHUANG)*
17:20- 18:20	**Junhui WANG** *(Chair: Xiaosheng ZHUANG)*
18:40	Welcome Reception - City Top Restaurant 9/F, Bank of China (Hong Kong) Complex, CityU

Time	Tuesday 19 Feb
9:00-10:00	**Amir AVERBUCH** *(Chair: Dany LEVIATAN)*
10:00-11:00	**Felipe CUCKER** *(Chair: Dany LEVIATAN)*
11:00-11:20	Coffee Break
11:20-12:20	**Elijah LIFLYAND** *(Chair: Dany LEVIATAN)*
12:30 -14:00	Free Lunch Time
14:00-15:00	**Patrick Tuen Wai NG** *(Chair: Dan DAI)*
15:00-16:00	**Shai DEKEL** *(Chair: Dan DAI)*
16:00-16:20	Coffee Break
16:20- 17:20	**Dan DAI** *(Chair: Patrick Tuen Wai NG)*
18:00	Workshop Banquet - House of Canton Shop 40, LG2, Festival Walk

Time	Wednesday 20 Feb
9:00-10:00	**Xianpeng HU** *(Chair: Junhui WANG)*
10:00-11:00	**Yoel SHKOLNISKY** *(Chair: Junhui WANG)*
11:00-11:20	Coffee Break
11:20-12:20	**Xiaosheng ZHUANG** *(Chair: Xianpeng HU)*
	Free Time

Time	Thursday 21 Feb
9:00-10:00	**Pierre NOLIN** *(Chair: Federick Weifeng QIU)*
10:00-11:00	**Nir SHARON** *(Chair: Federick Weifeng QIU)*
11:00-11:20	Coffee Break
11:20-12:20	**Federick Weifeng QIU** *(Chair: Pierre NOLIN)*
	Free Time

Geometric Component Analysis and its Applications to Data Analysis

AMIT BERMANIS, MOSHE SALHOV, AMIR AVERBUCH*
School of Computer Science, Tel Aviv University, Israel
Email: amir@math.tau.ac.il

Dimensionality reduction methods are designed to overcome the 'curse of dimensionality' phenomenon that makes the analysis of high dimensional big data difficult. Many of these methods are based on principal component analysis, which is statistically driven, and do not directly address the geometry of the data. Thus, machine learning tasks, such as classification and anomaly detection, may be affected for the bad.

This work provides a dictionary-based framework for geometrically driven data analysis, both for linear and diffusion (non-linear) geometries, that includes dimensionality reduction, out-of-sample extension and anomaly detection. The geometry of the data is preserved up to a user-defined distortion rate. In addition, a subset of landmark data points, known as dictionary, is identified by the presented algorithm. The performance of the method is demonstrated on both synthetic and real-world datasets. It achieves good results for unsupervised learning tasks.

Reconstructing Signals with Simple Fourier Transforms

HAIM AVRON*, MICHAEL KAPRALOV, CAMERON MUSCO, CHRISTOPHER MUSCO, AMEYA VELINGKER, AMIR ZANDIEH
School of Mathematical Sciences, Tel Aviv University, Israel
Email: haimav@post.tau.ac.il

Reconstructing continuous signals based on a small number of discrete samples is a fundamental problem across science and engineering. In practice, we are often interested in signals with "simple" Fourier structure – e.g., those involving frequencies within a bounded range, a small number of frequencies, or a few blocks of frequencies. More broadly, any prior knowledge about a signal's Fourier power spectrum can constrain its complexity. Intuitively, signals with more highly constrained Fourier structure require fewer samples to reconstruct.

We formalize this intuition by showing that, roughly speaking, a continuous signal from a given class can be approximately reconstructed using a number of samples equal to the statistical dimension of the allowed power spectrum of that class. We prove that, in nearly all settings, this natural measure tightly characterizes the sample complexity of signal reconstruction.

Surprisingly, we also show that, up to logarithmic factors, a universal non-uniform sampling strategy can achieve this optimal complexity for any class of signals. We present a simple, efficient, and general algorithm for recovering a signal from the samples taken. For bandlimited and sparse signals, our method matches the state-of-the-art. At the same time, it gives the first computationally and sample efficient solution to a broad range of problems, including multiband signal reconstruction and common kriging and Gaussian process regression tasks.

Our work is based on a novel connection between randomized linear algebra and the problem of reconstructing signals with constrained Fourier structure. We extend tools based on statistical leverage score sampling and column-based matrix reconstruction to the approximation of continuous linear operators that arise in the signal fitting problem. We believe that these extensions are of independent interest and serve as a foundation for tackling a broad range of continuous time problems using randomized methods.

Schlesinger transformations of invariant subspaces of biconfluent operators and Painlevé IV.

EDMUND YIK-MAN CHIANG
Department of Mathematics, Hong Kong University of Science and Technology, Hong Kong
Email: machiang@ust.hk

We show there is a correspondence between the parameter spaces of degenerate biconfluent Heun operators and degenerate Painlevé IV. Moreover, we show there are Schlesinger type transformations for classical scalar biconfluent Heun equations in terms of parabolic functions.

Computing the Homology of Semialgebraic Sets

Felipe Cucker
Department of Mathematics, City University of Hong Kong, Hong Kong
Email: macucker@cityu.edu.hk

We describe and analyze an algorithm for computing the homology (Betti numbers and torsion coefficients) of semialgebraic sets which works in weak exponential time. That is, outside of a set of exponentially small measure in the space of data the cost of the algorithm is exponential in the size of the data. All algorithms previously proposed for this problem have a complexity which is doubly exponential (and this is so for almost all data).

Gaussian unitary ensembles with pole singularities near the soft edge and a system of coupled Painlevé XXXIV equations

Dan Dai*, Shuai-Xia Xu, Lun Zhang
Department of Mathematics, City University of Hong Kong, Hong Kong
Email: dandai@cityu.edu.hk

In this paper, we study the singularly perturbed Gaussian unitary ensembles defined by the measure

$$\frac{1}{C_n} e^{-n \operatorname{tr} V(M;\lambda,\vec{t}\,)} dM,$$

over the space of $n \times n$ Hermitian matrices M, where $V(x;\lambda,\vec{t}\,) := 2x^2 + \sum_{k=1}^{2m} t_k (x-\lambda)^{-k}$ with $\vec{t} = (t_1, t_2, \ldots, t_{2m}) \in \mathbb{R}^{2m-1} \times (0,\infty)$, in the multiple scaling limit where $\lambda \to 1$ together with $\vec{t} \to \vec{0}$ as $n \to \infty$ at appropriate related rates. We obtain the asymptotics of the partition function, which is described explicitly in terms of an integral involving a smooth solution to a new coupled Painlevé system generalizing the Painlevé XXXIV equation. The large n limit of the correlation kernel is also derived, which leads to a new universal class built out of the Ψ-function associated with the coupled Painlevé system.

Numerical PDEs say hello to my little ML friend!

SHAI DEKEL*, DAN GIVOLI, ADAR KAHANA, ELI TURKEL
School of Mathematical Sciences, Tel Aviv University, Israel
Email: shaidekel6@gmail.com

We believe that the field of numerical solutions of partial differential equations is on the verge of being revolutionized through modern machine learning in the same manner as the fields of computer vision and natural language processing. We anticipate that through training on many software simulations, solvers based on deep neural network architectures will be able to surpass in performance existing methods and even provide solutions in ill-posed scenarios where existing inverse problem methods fail.

As an example, we will review the problem of the wave equation where a finite number of scattered sensors collect noisy time series data and the goal is to find the unknown location of the source or the location and geometry of unknown obstacles.

Wellposedness of elastic fluid flows

XIANPENG HU
Department of Mathematics, City University of Hong Kong, Hong Kong
Email: xianpehu@cityu.edu.hk

In this talk, we will discuss some recent progress in the mathematical analysis for the complex fluids, especially elastic fluid flows. Both the incompressible case and the compressible case are considered. Global existences of either strong solutions or weak solutions are two main subjects.

Are the degrees of unconstrained and constrained approximation by polynomials the same?

D. Leviatan
School of Mathematical Sciences, Tel Aviv University, Israel
Email: leviatan@tauex.tau.ac.il

It is quite obvious that one should expect that the degree of constrained approximation be worse than the degree of unconstrained approximation. However, it turns out that in certain cases we can deduce the behavior of the degrees of the former from information about the latter.

Let $E_n(f)$ denote the degree of approximation of $f \in C[-1,1]$, by algebraic polynomials of degree $< n$, and assume that we know that for some $\alpha > 0$ and $N \geq 1$,

$$n^\alpha E_n(f) \leq 1, \quad n \geq N.$$

Suppose that $f \in C[-1,1]$, changes its monotonicity or convexity $s \geq 0$ times in $[-1,1]$ ($s = 0$ means that f is monotone or convex, respectively). We are interested in what may be said about its degree of approximation by polynomials of degree $< n$ that are comonotone or coconvex with f. Specifically, if f changes its monotonicity or convexity at $Y_s := \{y_1, \dots, y_s\}$ ($Y_0 = \emptyset$) and the degrees of comonotone and coconvex approximation are denoted by $E_n^{(q)}(f, Y_s)$, $q = 1, 2$, respectively. We investigate when can one say that

$$n^\alpha E_n^{(q)}(f, Y_s) \leq c(\alpha, s, N), \quad n \geq N^*,$$

for some N^*. Clearly, N^*, if it exists at all (we prove it always does), depends on α, s and N. However, it turns out that for certain values of α, s and N, N^* depends also on Y_s and, in some cases, even on f itself and this dependence is essential.

Fixed-Point Theory for Trees of Mappings

DAVID LEVIN
School of Mathematical Sciences, Tel Aviv University, Israel
Email: levindd@gmail.com

Both in subdivision methods and in fractal analysis, geometrical objects are generated by iterative procedures. Motivated by both subjects we consider a general framework for presenting mapings, namely, trees of mapings. we define iterated procedures on such trees and investigate their convergence. The main tool here is the analysis of backward trajectories presented in an earlier work. Trees of mapings can represent different kinds of subdivision processes, from stationary uniform subdivision, to non-stationary subdivision, and also non-uniform and non-linear subdivision. In the fractals' perspective, such trees of mappings suggest new types of fractals generated by code-dependent function systems.

Joint work with Nira Dyn and Peter Massopust

The Fourier transform of a convex function

ELIJAH LIFLYAND
Department of Mathematics, Bar-Ilan University, Ramat-Gan, Israel
Email: liflyand@gmail.com

A variety of new results on the Fourier transform of a function of bounded variation has appeared in the last 25 years. Functions of bounded variation are of extreme importance and continuing use in many fields of mathematics and in numerous applications. Let us mention the moment problem and harmonic analysis in dimension one and calculus of variations in several dimensions. Many of integrability conditions originated from the noteworthy result of Trigub on the asymptotic behavior of the Fourier transform of a convex function. This result is in turn a generalization of the old result by Shilov on the asymptotic behavior of the Fourier coefficients of a convex function. However, convex functions still remain to be one of the important classes in many problems. First of all, they play an essential role in convex geometry and convex optimization, but also in harmonic analysis they continuously are on the scene. New asymptotic-wise results for the Fourier transform of a function of convex type are discussed. More precisely, the main idea is to get an explicit expression for the remainder terms in the point-wise relation for the Fourier transform, handy enough and controllable, at least for convex functions. Next, which is a consequence of such a precise formula, we generalize the obtained results to the multidimensional case. Surprisingly, though many of the results on the Fourier transform of a function of bounded variation were extended to several dimensions, the basic results with convexity were generalized only in very particular cases, like say for radial functions. The reason apparently was in the lack of appropriate language. We introduce a new notion of the so-called robust convexity. Along with the notion of balanced operators it gives perfect tools for obtaining far-going multivariate generalizations of the known and new one-dimensional results.

Polynomials versus finite Blaschke products

TUEN WAI NG
Department of Mathematics, The University of Hong Kong, Hong Kong
Email: ntw@maths.hku.hk

It is well known that the only proper holomorphic self maps of the complex plane are polynomials. In 1923, Fatou proved that finite Blaschke products are the only proper holomorphic self maps of the standard unit disk. In this talk, we will further compare polynomials and finite Blaschke products and demonstrate that they share many other similar properties. This allows us to establish a dictionary between polynomials and finite Blaschke products.

For example, we will present a version of Ritt's theory on the factorization (in the sense of composition) of finite Blaschke products. In this Ritt's theory on the unit disk, a special class of finite Blaschke products has been introduced as the counterpart of Chebyshev polynomials in Ritt's theory for polynomials. These special finite Blaschke products are therefore called Chebyshev-Blaschke products. In this talk, I will explain the construction of them and also discuss some of their interesting properties related to approximation theory and number theory.

Self-organized criticality in 2D forest fire processes

PIERRE NOLIN
Department of Mathematics, City University of Hong Kong, Hong Kong
Email: bpmnolin@cityu.edu.hk

Bernoulli percolation is a model for random media introduced by Broadbent and Hammersley in 1957. In this process, each vertex of a given graph is occupied or vacant, with respective probabilities p and $1-p$, independently of the other vertices (for some parameter p). It is arguably one of the simplest models from statistical mechanics displaying a phase transition as the parameter p varies, i.e. a drastic change of behavior at some critical value p_c, and it has been widely studied.

Percolation can be used to analyze forest fire processes on a two-dimensional lattice. In such processes, all vertices are initially vacant, and then become occupied at rate 1. If an occupied vertex is hit by lightning, which occurs at a (typically very small) rate, its entire occupied cluster burns immediately (all its vertices become vacant). In particular, we want to analyze the near-critical behavior of such processes, that is, when large connected components of occupied sites start to appear. They display a form of self-organized criticality, and the phase transition of Bernoulli percolation plays an important role: it appears "spontaneously".

This talk is based on a joint work with Rob van den Berg (CWI and VU, Amsterdam).

Analysis of a mixed finite element method for the quad-curl problem

QIU FEDERICK WEIFENG
Department of Mathematics, City University of Hong Kong, Hong Kong
Email: weifeqiu@cityu.edu.hk

Quad-curl term plays an essential role in the numerical analysis of the resistive magnetohydrodynamics (MHD) and the forth order inverse electromagnetic scattering problem. It is desirable to develop simple and efficient numerical methods for the quad-curl problem. In this paper, we firstly give a regularity result for the quad-curl problem on Lipschitz polyhedron domains, which is new in literatures. Then, we propose a mixed finite element method for the quad-curl problem. With novel discrete Sobolev embedding inequalities for the piecewise polynomials, we obtain stability results and derive optimal error estimates relying on a low regularity assumption of the exact solution. To the best of our knowledge, this low regularity assumption is lower than the regularity requirements in existing works.

The big bang theory of multivariate splines

Amos Ron
Department of Computer Sciences, University of Wisconsin, USA
Email: amos@cs.wisc.edu

At the beginnings, the vibrant universe of multivariate splines did not exist. There was just one **graph**, a finite graph, whose edges continuously flipped their directions in a random way. From time to time, like a good slot machine in Las Vegas, all the orientations of the edges clicked into a harmonious configuration, and small nuclei-like shape known as **parking function** bursted in the chaotic vacuum and began orbiting the graph. It was William Tutte who was able to count the number of parking functions emitted by a graph: a complicated number in general, but sometimes simple: $n!$ for a complete graph with $n+1$ vertices, for example.

According to the **Big Bang Theory** of multivariate splines, one day the big bang occurred and the parking functions, each, exploded.

On the negative side, anti-matter was created. Its destructive force was culminated by the thunder power of the **torsion ideal**. The torsion ideal was discovered in the 80's by a group of scientists led by Carl de Boor, Ron DeVore, Klaus Höllig and others. It received its name from its role in Jeffrey-Kirwan (JK) decompositions a la Brion-Vergne. One thunder ball from the torsion ideal destroys all matter. Below the torsion ideal rests the evil empire of the $\mathcal{P}$-polynomials. Those were discovered a few years later by Dahmen and Micchelli, and by Dyn and Ron. They destroy matter partially, sending some matter to dust, and reducing other to more rudimentary form. In JK decompositions they are known as "free", but make no mistake about these guys and their destructive power. The evolution of the evil empire from the parking functions is well understood, is not too complicated, and goes under the umbrella of "monomization of power ideals". It is well documented in the work of Postnikov, Shapiro and others in the 00's.

On the positive side there is matter. The small building blocks of matter are the $\mathcal{D}$-polynomials. These were thought to be complex objects beyond imagination, with magic powers of combining themselves, in a smooth piecewise-manner, to create wonderful complicated constructs known as *truncated powers* and *partition functions.* These truncated powers and partitions functions then spread themselves over the science universe, helping the mortals in their daily tasks of computing volumes, counting number of solutions, representing Schur functions and moment maps, building the even more complex guys known as box splines, and much, much more. Matter was introduced by the same people who introduced the torsion ideal: at the end, the only power of the torsion ideal is to destroy matter, it can destroy nothing else! The torsion ideal kills the truncated power all the way down to hyperplane dust, which occupies no volume. The evil empire is not so powerful, as the $\mathcal{P}$-polynomials destroy the truncated power into flat constants. Matter can be classified by duality according to the action of the $\mathcal{P}$-polynomials. But the intrinsic meaning of that classification has never been understood, and the duality does not help in understanding the shape and form of matter.

We complete in this talk the Big Bang Theory of multivariate splines, by showing that matter also evolved from the same parking functions, and describe exactly how that evolution worked. So, each parking function led to the creation of one atom of matter, crowned as **flow polynomial** and one dual evil-atom of anti-matter, creating a universe of basis of atoms and dual basis of evil-atoms. It turns out that the evolution of matter from the parking functions all the way to the flow polynomials is simple and concrete, making matter simpler to understand than anti-matter!. The destructive nature of the evil-atoms on matter, became a simple task of monitoring orientations in directed graphs, a task that all mortals of the universe (even those known as undergraduate students) can easily perform. And finally, the assembly of truncated powers from their matter, i.e., from flow polynomials, becomes a simple task: it is just a simple interaction between matter and anti-matter that makes it happen.

The reported work is a joint endeavor with Shengnan (Sarah) Wang.

Current Advances in subdivision schemes on manifolds

Nira Dyn, Nir Sharon*
School of Mathematical Sciences, Tel Aviv University, Israel
Email: nsharon@tauex.tau.ac.il

Recent years gave rise to exciting developments in methods for approximating manifolds and manifold-valued objects. In this talk, we focus on the case of one-dimensional data series over manifolds and the use of univariate subdivision schemes for generating smooth curves on the manifold, starting from the data series. In particular, we concentrate on two fundamental questions: how can we construct such operators and their analysis.

Manifold methods for image denoising in cryo-electron microscopy

YOEL SHKOLNISKY*, BORIS LANDA
Department of Applied Mathematics, School of Mathematical Sciences, Tel-Aviv University, Israel
Email: yoelsh@tauex.tau.ac.il

The Nobel Prize in Chemistry 2017 was awarded "for developing cryo-electron microscopy for the high-resolution structure determination of biomolecules in solution". In a nutshell, cryo-electron microscopy is a method to determine the three-dimensional structure of molecules from their two-dimensional images taken by an electron microscope. A key challenge in cryo-electron microscopy is that the two-dimensional images are extremely noisy, and must be denoised first to improve their quality (signal-to-noise ratio).

We present a method for denoising the two-dimensional images that exploits the fact that all (unknown) noiseless images lie on an (unknown) three-dimensional manifold. The method is based on constructing a "Fourier-like" basis on the manifold, followed by using a subset of the basis functions to "filter" the images. We show that all the quantities required by the filtering procedure can be estimated using only the input two-dimensional images. Moreover, the proposed approach allows using not only the given input images, but also all their infinitely many in-plane rotations.

A smooth collaborative recommender system

JUNHUI WANG
Department of Mathematics, City University of Hong Kong, Hong Kong
Email: j.h.wang@cityu.edu.hk

In recent years, there has been a growing demand to develop efficient recommender systems which track users' preferences and recommend potential items of interest to users. In this talk, I will present a smooth collaborative recommender system to utilize dependency information among users and items which share similar characteristics under the singular value decomposition framework. The proposed method incorporates the neighborhood structure among user-item pairs by exploiting covariates to improve the prediction performance. One key advantage of the proposed method is that it leads to more effective recommendation for "cold-start" users and items, whose preference information is completely missing from the training set. As this type of data involves large-scale customer records, efficient scheme will be proposed to achieve scalable computing. The advantage is confirmed in a variety of simulated experiments as well as one large-scale real example on *Last.fm* music listening counts. If time permits, the asymptotic properties will also be discussed.

Theory of Deep Convolutional Neural Networks for Deep Learning

DING-XUAN ZHOU
School of Data Science and Department of Mathematics, City University of Hong Kong, Hong Kong
Email: mazhou@cityu.edu.hk

Deep learning has been widely applied and brought breakthroughs in speech recognition, computer vision, and many other domains. The involved deep neural network architectures and computational issues have been well studied in machine learning. But there lacks a theoretical foundation for understanding the approximation or generalization ability of deep learning methods with network architectures such as deep convolutional neural networks (CNNs) with convolutional structures. The convolutional architecture gives essential approximation theory differences between the deep CNNs and fully-connected deep neural networks, and the classical approximation theory of fully-connected networks developed around 30 years ago does not apply. This talk describes an approximation theory of deep CNNs. In particular, we show the universality of a deep CNN, meaning that it can be used to approximate any continuous function to an arbitrary accuracy when the depth of the neural network is large enough. Our quantitative estimate, given tightly in terms of the number of free parameters to be computed, verifies the efficiency of deep CNNs in dealing with large dimensional data. Some related distributed learning algorithms will also be discussed.

Directional Framelets and Applications

XIAOSHENG ZHUANG
Department of Mathematics, City University of Hong Kong, Hong Kong
Email: xzhuang7@cityu.edu.hk

Directional multiscale representation systems play an important role in both theory and applications. In theory, directional systems such as curvelets and shearlets have been proved to provide (nearly) optimal approximate rate for cartoon-like functions. In applications such as image/video denoising/inpainting, directional systems have been shown to outperform classical tensor product real-value wavelet/framelet systems. In this talks, we will discuss the design and applications of several types of directional multiscale representation systems, including

1) directional Haar tight framelets and their induced directional box spline tight framelets;

2) tensor product complex tight framelets;

3) affine shear tight frames and affine shear frames with 2-layer structure.