Course Syllabus

offered by Department of Mathematics
with effect from Semester A 2022/23

Part I Course Overview

Course Title:	**Selected Topics in Applied Analysis**
Course Code:	**MA8004**
Course Duration:	**One semester**
Credit Units:	**3**
Level:	**R8**
Medium of Instruction:	**English**
Medium of Assessment:	**English**
Prerequisites: *(Course Code and Title)*	**Nil**
Precursors: *(Course Code and Title)*	**Nil**
Equivalent Courses: *(Course Code and Title)*	**Nil**
Exclusive Courses: *(Course Code and Title)*	**Nil**

Part II Course Details

1. Abstract

This course aims to introduce research students to three active fields of research about partial differential equations depending on a parameter, which may go either to zero or to plus infinity, namely:

- Asymptotic analysis
- Homogenization
- Penalization

This will be done by studying four fundamental boundary value problems in applied mathematics, which also have numerous real-life applications:

1. Stokes equations: existence, uniqueness, and asymptotic analysis in infinite cylinders,
2. Elasticity equations: existence, uniqueness, and asymptotic analysis in thin plates,
3. Diffusion equation: existence, uniqueness, and homogenization,
4. Obstacle problems: existence, uniqueness, and penalization.

The course is self-contained, save for basic theorems about Sobolev spaces.

2. Course Intended Learning Outcomes (CILOs)

No.	CILOs[#]	Weighting* (if applicable)	Discovery-enriched curriculum related learning outcomes (please tick where appropriate)		
			A1	*A2*	*A3*
1.	Define weak, strong, and classical solutions for any boundary value problem made of partial differential equations and boundary conditions	10%	✓	✓	
2.	Prove existence, uniqueness and stability of weak solutions to boundary value problems of elliptic type, by using Riesz/Lax Milgram theorem	10%	✓	✓	
3.	Find the limit of the weak solution to a boundary value problem depending on a parameter, by using the "corrector method" taught in Chapter 1 of this course	20%		✓	✓
4.	Find the limit of the solution of a boundary value problem depending on a parameter by using the "scaling method" taught in Chapter 2	20%		✓	✓
5.	Find the homogenized problem corresponding to a PDE with highly oscillating coefficients by using the technique taught in Chapter 3	20%	✓	✓	✓
6.	Compute the solution of a constrained problem by using the penalization method taught in Chapter 4	20%	✓	✓	✓
		100%			

A1: *Attitude*
Develop an attitude of discovery/innovation/creativity, as demonstrated by students possessing a strong sense of curiosity, asking questions actively, challenging assumptions or engaging in inquiry together with teachers.

A2: *Ability*
Develop the ability/skill needed to discover/innovate/create, as demonstrated by students possessing critical thinking skills to assess ideas, acquiring research skills, synthesizing knowledge across disciplines

or applying academic knowledge to self-life problems.

A3: *Accomplishments*
Demonstrate accomplishment of discovery/innovation/creativity through producing /constructing creative works/new artefacts, effective solutions to real-life problems or new processes.

3. Teaching and Learning Activities (TLAs)

TLA	Brief Description	CILO No.						Hours/week (if applicable)
		1	2	3	4	5	6	
Lectures	Learning through teaching is primarily based on lectures	✓	✓	✓	✓	✓	✓	3 hours/week
Assignments	Learning though take-home assignments helps students understand how to apply the techniques learned in this course to other boundary value problems modeling physical phenomena	✓	✓	✓	✓	✓	✓	After class

4. Assessment Tasks/Activities (ATs)

Assessment Tasks/Activities	CILO No.						Weighting*	Remarks
	1	2	3	4	5	6		
Continuous Assessment: 50%								
Test	✓	✓	✓	✓			25%	Questions are designed for the first part of the course to see how well students have learned to prove existence, uniqueness, and stability of weak solutions to a boundary value problem of elliptic type.
Hand-in assignments	✓	✓	✓	✓	✓	✓	25%	These are skills based assessment to help students learn how to apply the techniques learned in this course to various mathematical problems modeling physical phenomena.
Examination: 50% (duration: 3 hours)	✓	✓	✓	✓	✓	✓	50%	Examination questions are designed to see how far students have achieved their intended learning outcome. Questions will primarily be skills and understanding based to assess the student’s versatility in techniques used to tackle boundary values problems depending on a parameter.
							100%	

5. Assessment Rubrics

Applicable to students admitted in Semester A 2022/23 and thereafter

Assessment Task	Criterion	Excellent (A+, A, A-)	Good (B+, B)	Marginal (B-,C+,C)	Failure (F)
1. Test	Demonstration of the understanding of the first part of the course	High	Significant	Basic	Not even reaching marginal levels
2. Hand in assignments	Demonstration of the understanding of the basic techniques taught in this course	High	Significant	Basic	Not even reaching marginal levels
3. Examination	Demonstration of the skills and versatility in applied analysis	High	Significant	Basic	Not even reaching marginal levels

Applicable to students admitted before Semester A 2022/23

Assessment Task	Criterion	Excellent (A+, A, A-)	Good (B+, B, B-)	Fair (C+, C, C-)	Marginal (D)	Failure (F)
1. Test	Demonstration of the understanding of the first part of the course	High	Significant	Moderate	Basic	Not even reaching marginal levels
2. Hand in assignments	Demonstration of the understanding of the basic techniques taught in this course	High	Significant	Moderate	Basic	Not even reaching marginal levels
3. Examination	Demonstration of the skills and versatility in applied analysis	High	Significant	Moderate	Basic	Not even reaching marginal levels

Part III Other Information (more details can be provided separately in the teaching plan)

1. Keyword Syllabus

Asymptotic analysis, Homogenization, Penalization; weak solutions to partial differential equations, Riesz theorem and its generalization by Lax-Milgram, scaling, a priori estimates; Stokes equations, Elasticity equations, Diffusion problem, Obstacle problem.

2. Reading List

2.1 Compulsory Readings

1.	
2.	
3.	
…	

2.2 Additional Readings

1.	Michel M. CHIPOT: *Asymptotic Issues for some Partial Differential Equations.* Imperial College Press, 2016.
2.	Philippe G. CIARLET: *Mathematical Elasticity, Vol. II: Theory of Plates*. North-Holland, 1997.
3.	Doina CIORANESCU, Alain DAMLAMIAN, Georges GRISO: *The Periodic Unfolding Method. Theory and Applications to Partial Differential Problems.* Springer, 2018.
4.	David KINDERLEHRER, Guido STAMPACCHIA: *An Introduction to Variational Inequalities and Their Applications.* SIAM, 2000.