# City University of Hong Kong Course Syllabus

# offered by College/School/Department of <u>Mathematics</u> with effect from Semester <u>B 2024/25</u>

# Part I Course Overview

Course Title:	Stochastic Interest Rate Models
Course Code:	MA6627
<b>Course Duration:</b>	One Semester
Credit Units:	3 CUs
Level:	P6
Medium of	English
mști uction.	
Medium of	English
Assessment.	
Prerequisites:	MA5616 Financial Mathematics in Davinative Markets
(Course Code and Title)	MASolo Financial Mathematics in Derivative Markets
Precursors:	Nil
(Course Code and Title)	
<b>Equivalent Courses</b> :	NT21
(Course Code and Title)	
Exclusive Courses:	
(Course Code and Title)	NII

## Part II Course Details

#### 1. Abstract

This course aims to

- examine quantitative aspects of interest rate models and pricing of associated derivatives, such as caps and swaps;
- present calibration methods to stochastic interest rate models, including short rate and forward rate models; and
- introduce modeling of stochastic term structure models and hedging from an infinite-dimensional viewpoint.

## 2. Course Intended Learning Outcomes (CILOs)

(CILOs state what the student is expected to be able to do at the end of the course according to a given standard of performance.)

No.	CILOs	Weighting (if applicable)	Discov curricu learnin (please approp	very-en ilum rel ig outco e tick priate)	riched lated omes where
			A1	A2	A3
1.	Describe short rate models by Itô stochastic processes and derive governing equations of bond prices with no-arbitrage argument.	40%	$\checkmark$	~	
2.	Model stochastic movement of forward rates with HJM model.	20%	~	~	
3.	Formulate LIBOR market model (BGM model) of interest rate derivatives under the Gaussian HJM framework.	20%	~	~	
4.	Apply the technique of forward measure to price interest rate derivatives and to examine expectation of the short rate and the LIBOR process.	10%	$\checkmark$	~	~
5.	Present an infinite-dimensional analysis to interest rate term structure and hedging derivatives.	10%	$\checkmark$	$\checkmark$	$\checkmark$
		100%			

A1: Attitude

Develop an attitude of discovery/innovation/creativity, as demonstrated by students possessing a strong sense of curiosity, asking questions actively, challenging assumptions or engaging in inquiry together with teachers.

A2: Ability

Develop the ability/skill needed to discover/innovate/create, as demonstrated by students possessing critical thinking skills to assess ideas, acquiring research skills, synthesizing knowledge across disciplines or applying academic knowledge to real-life problems.

A3: Accomplishments

Demonstrate accomplishment of discovery/innovation/creativity through producing /constructing creative works/new artefacts, effective solutions to real-life problems or new processes.

## 3. Learning and Teaching Activities (LTAs)

(LTAs designed to facilitate students' achievement of the CILOs.)

LTA	Brief Description		O No.			Hours/week	
		1	2	3	4	5	(if applicable)
Lectures	Students will engage in lecture activities about various stochastic interest rate models.	~	~	~	~	$\checkmark$	39 hours in total
Assignments	Students are required to finish <b>take-home assignments</b> which helps them characterize and analyze various interest rate models with stochastic techniques, as well as apply these models in pricing traded interest rate derivatives.	V	V	V	V	$\checkmark$	after-class

#### 4. Assessment Tasks/Activities (ATs)

(ATs are designed to assess how well the students achieve the CILOs.)

30% Coursework

70% Examination (Duration: 3 hours, at the end of the semester)

For a student to pass the course, at least 30% of the maximum mark for the examination must be obtained.

Assessment Tasks/Activities	CILO No.				Weighting	Remarks		
	1	2	3	4	5	6		
Continuous Assessment:30%								
Test	~	~	✓ 				20%	Questions are designed for the first part of the course to see how well students have learned stochastic analysis of interest rate models, such as HJM and LIBOR models.
Hand-in assignments	~	✓	~	~	✓		10%	These are skills-based assessment which enables students to model interest rate and forward rate with stochastic methods, as well as to formulate HJM framework and LIBOR processes.
Examination: 70% (duration:	3 hrs	, if a <sub>j</sub>	pplic	able)				Examination questions are designed to see how far students have achieved their intended learning outcomes. Questions will primarily be skills and understanding based to assess the student's versatility in mathematical methods underlying stochastic interest rate models and associated derivatives pricing.
							100%	

# 5. Assessment Rubrics

(Grading of student achievements is based on student performance in assessment tasks/activities with the following rubrics.)

#### Applicable to students admitted before Semester A 2022/23 and in Semester A 2024/25 & thereafter

Assessment Task	Criterion	Excellent	Good	Fair	Marginal	Failure
		(A+, A, A-)	(B+, B, B-)	(C+, C, C-)	(D)	(F)
1. Test		Demonstrates a	Demonstrates a	Demonstrates a	Demonstrates a	Demonstrates a little
		thorough	substantial	general	partial understanding	concepts and
		understanding of	understanding of the	understanding of	of the concepts and	techniques in the
	Independent problem-	the concepts and	concepts and	the concepts and	techniques in the	rate theory and can
	solving skills on	techniques in the	techniques in the	techniques in the	stochastic interest	rarely or never apply
	stochastic analysis of interest rate models.	stochastic interest	stochastic interest	stochastic interest	rate theory and can	the techniques to solve HJM and
	including HJM and	rate theory and can	rate theory and can	rate theory and can	seldom apply the	LIBOR models.
	LIBOR models	always apply the	usually apply the	sometimes apply	techniques to solve	
	techniques to solve	techniques to solve	the techniques to	HJM and LIBOR		
		HJM and LIBOR	HJM and LIBOR	solve HJM and	models.	
		models.	models.	LIBOR models.		
2. Hand-in		Demonstrates a	Demonstrates a	Demonstrates a	Demonstrates a	Demonstrates a little
assignments	thorough	substantial	substantial	partial understanding	stochastic method to	
	Understanding of the		understanding of the	understanding of	of the stochastic	model interest rate
	stochastic method to model interest rate and forward rate, as well as to formulate HJM	the stochastic	stochastic method to	the stochastic	method to model	and forward rate and can rarely or never
		method to model	model interest rate	method to model	interest rate and	apply the method to
		interest rate and	and forward rate and	interest rate and	forward rate and can	formulate HJM framework and
	framework and LIBOR	forward rate and	can usually apply the	forward rate and	seldom apply the	LIBOR processes.
	processes	can always apply	method to formulate	can usually apply	method to formulate	
		the method to	HJM framework and	the method to	HJM framework and	
		formulate HJM	LIBOR processes.	formulate HJM	LIBOR processes.	

		framework and		framework and		
		LIBOR processes.		LIBOR processes.		
3. Examination		Demonstrates a	Demonstrates a	Demonstrates a	Demonstrates a	Demonstrates a little
		thorough	substantial	substantial	partial understanding	mathematical theory
		understanding of	understanding of the	understanding of	of the mathematical	on stochastic interest
		the mathematical	mathematical theory	the mathematical	theory on stochastic	rate models and associated
		theory on	on stochastic interest	theory on	interest rate models	derivatives pricing
		stochastic interest	rate models and	stochastic interest	and associated	and can rarely or never apply the
	Comprehensive	rate models and	associated	rate models and	derivatives pricing	theory to solve
	on mathematical	associated	derivatives pricing	associated	and can seldom apply	problems on
	methods underlying	derivatives pricing	and can usually	derivatives pricing	the theory to solve	rate models and
	stochastic interest rate	and can always	apply the theory to	and can usually	problems on	associated
	derivatives pricing	apply the theory to	solve problems on	apply the theory to	stochastic interest	derivatives pricing.
		solve problems on	stochastic interest	solve problems on	rate models and	
		stochastic interest	rate models and	stochastic interest	associated derivatives	
		rate models and	associated	rate models and	pricing.	
		associated	derivatives pricing.	associated		
		derivatives		derivatives pricing.		
		pricing.				

Applicable to students admitted from Semester A 2022/23 to Summer Term 2024
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Assessment Task	Criterion	Excellent	Good	Marginal	Failure
		(A+, A, A-)	(B+, B)	(B-, C+, C)	(F)
1. Test	Independent problem-solving	Demonstrates a thorough understanding of the concepts and techniques in the	Demonstrates a substantial understanding of the concepts and techniques	Demonstrates a general understanding of the concepts and techniques in the stochastic interest	Demonstrates a little understanding of the concepts and techniques in the stochastic interest
	skills on stochastic analysis of interest rate models, including HJM and LIBOR models	stochastic interest rate theory and can always apply the techniques to solve HJM and LIBOR models.	in the stochastic interest rate theory and can usually apply the techniques to solve HJM and LIBOR models.	rate theory and can sometimes apply the techniques to solve HJM and LIBOR models.	rate theory and can rarely or never apply the techniques to solve HJM and LIBOR models.
2. Hand-in assignments	Understanding of the stochastic method to model interest rate and forward rate, as well as to formulate HJM framework and LIBOR processes	Demonstrates a thorough understanding of the stochastic method to model interest rate and forward rate and can always apply the method to formulate HJM framework and LIBOR processes.	Demonstrates a substantial understanding of the stochastic method to model interest rate and forward rate and can usually apply the method to formulate HJM framework and LIBOR processes.	Demonstrates a general understanding of the stochastic method to model interest rate and forward rate and can sometimes apply the method to formulate HJM framework and LIBOR processes.	Demonstrates a little understanding of the stochastic method to model interest rate and forward rate and can rarely or never apply the method to formulate HJM framework and LIBOR processes.
3. Examination	Comprehensive problem- solving skills on mathematical methods underlying stochastic interest rate models and associated derivatives pricing	Demonstrates a thorough understanding of the mathematical theory on stochastic interest rate models and associated derivatives pricing and can always apply the theory to solve problems on stochastic interest rate models and associated derivatives pricing.	Demonstrates a substantial understanding of the mathematical theory on stochastic interest rate models and associated derivatives pricing and can usually apply the theory to solve problems on stochastic interest rate models and associated derivatives pricing.	Demonstrates a general understanding of the mathematical theory on stochastic interest rate models and associated derivatives pricing and can sometimes apply the theory to solve problems on stochastic interest rate models and associated derivatives pricing.	Demonstrates a little understanding of the mathematical theory on stochastic interest rate models and associated derivatives pricing and can rarely or never apply the theory to solve problems on stochastic interest rate models and associated derivatives pricing.

Part III Other Information (more details can be provided separately in the teaching plan)

## 1. Keyword Syllabus

(An indication of the key topics of the course.)

Short term interest rate models. Zero-coupon bonds. Forward rates. Term structure dynamics-HJM model. Arbitrage and change of numeraire. LIBOR market-BGM model. Interest rate derivatives. Pricing of caps and swaps. Infinite dimensional approach to hedging. Calibration issues.

## 2. Reading List

## 2.1 Compulsory Readings

(Compulsory readings can include books, book chapters, or journal/magazine articles. There are also collections of e-books, e-journals available from the CityU Library.)

1.	Steven E. Shreve, Stochastic Calculus for Finance I: The Binomial Asset Pricing Model,
	Springer; 2004th edition
2.	Steven E. Shreve, Stochastic Calculus for Finance II: Continuous-Time Models, Springer,
	2010
3.	

## 2.2 Additional Readings

(Additional references for students to learn to expand their knowledge about the subject.)

1.	
2.	
3.	